Interplay of Disorders and Nonlinearities in Spin Seebeck Effect

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OUTLINE

◆Introduction

- ✓ Spin Seebeck effect
- ✓ Nonequilibrium steady state (NESS)

◆Energy repartition in NESS: Magnon

- ✓ Mode temperature for magnon
- ✓ Renormalized dispersion

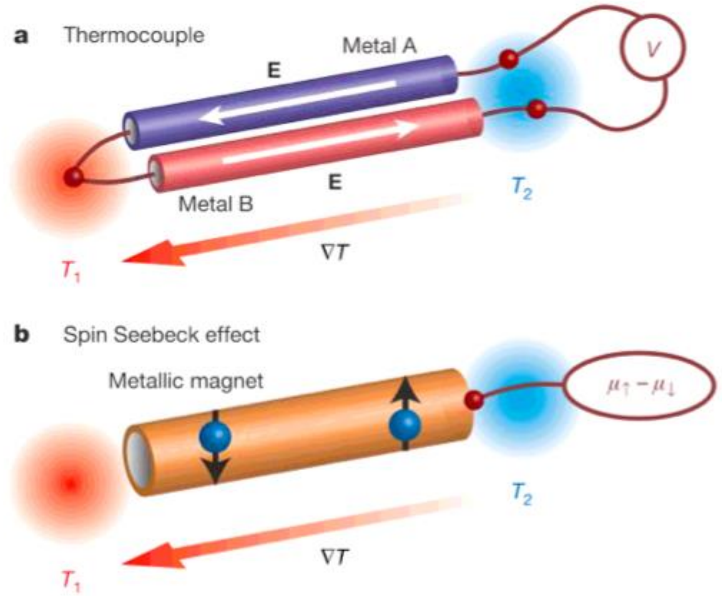
◆Wave localization and spin Seebeck effect

- ✓ Spin wave localization in spin Seebeck effect
- ✓ Nonlinearity

◆Discussion

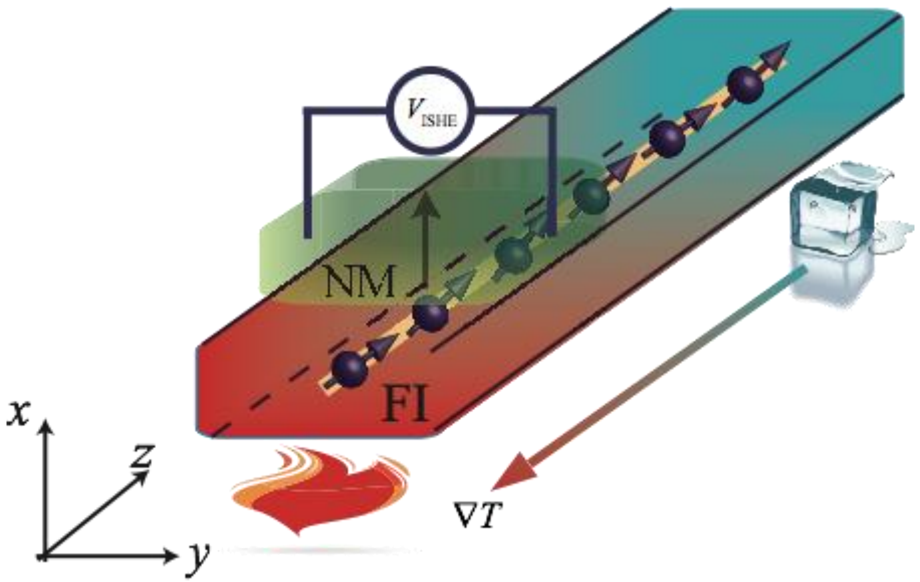
SPIN SEEBECK EFFECT

Spin ⟷ heat



K. Uchida *et al*, NATURE **455**, 778, 2008

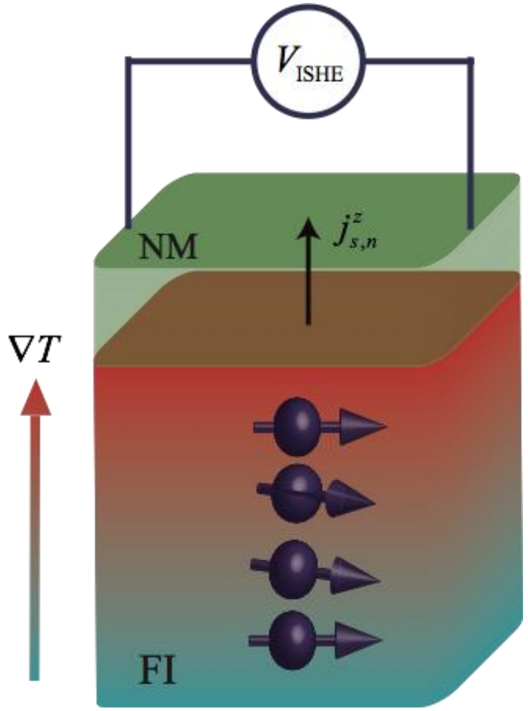
Transverse configuration



K. Uchida *et al*, NATURE MATERIALS **9**, 894, 2010

C. M. Jaworski *et al*, NATURE MATERIALS **9**, 898, 2010

Longitudinal configuration



K. Uchida *et al*, APL **97**, 172505, 2010

M. Weiler *et al*, PRL **108**, 106602, 2012

K. Uchida *et al*, PRX **4**, 041023, 2014

TRANSVERSE SPIN SEEBECK EFFECT

Report of TSSE	Report of NO TSSE
K. Uchida *et al*, NATURE **455**, 778, 2008	S. Y. Huang *et al*, PHYSICAL REVIEW LETTERS **107**, 216604, 2011
K. Uchida *et al*, NATURE MATERIALS **9**, 894, 2010	M. Schmid *et al*, PHYSICAL REVIEW LETTERS **111**, 187201, 2013
C. M. Jaworski *et al*, NATURE MATERIALS **9**, 898, 2010	D. Meier *et al*, PHYSICAL REVIEW B **88**, 184425, 2013
R. McLaughlin *et al*, PHYSICAL REVIEW B **95**, 180401, 2017	C. T. Bui *et al*, PHYSICAL REVIEW B **90**, 100403, 2014
	D. Meier *et al*, NATURE COMMUNICATIONS, **6**, 8211, 2015

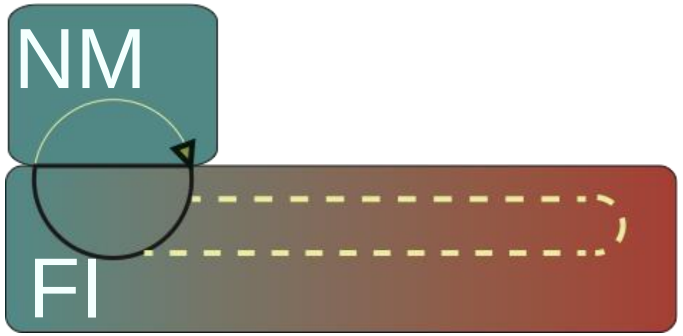
J. Xiao *et al*, PRB **81**, 214418, 2010

H. Adachi *et al*, PRB **83**, 094410, 2011

Local thermal equilibrium

Sample length

Bar length ~ 0.1 mm



H. Adachi *et al*, APL **97**, 252506, 2010

C. M. Jaworski *et al*, PRL**106**, 186601, 2011

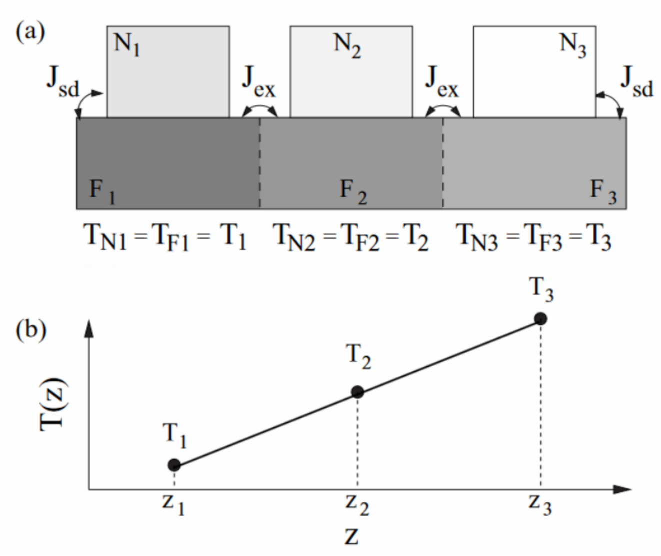
Two essential issues:

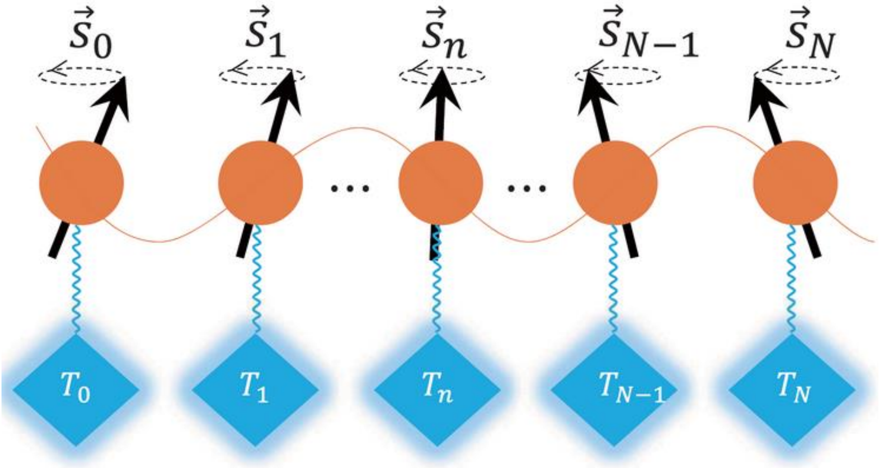
Does TSSE exist in FI (magnon)?

If it exists, why the spin current change sign?

LOCAL THERMAL EQUILIBRIUM

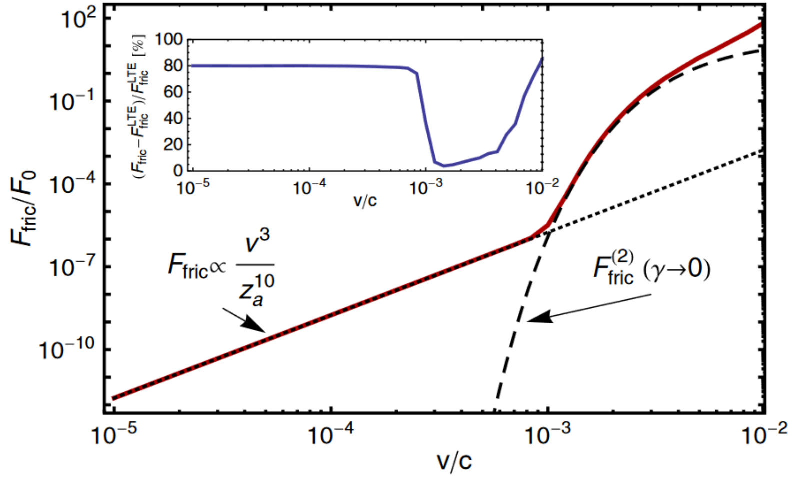
H. Adachi et al, PRB **83**, 094410, 2011





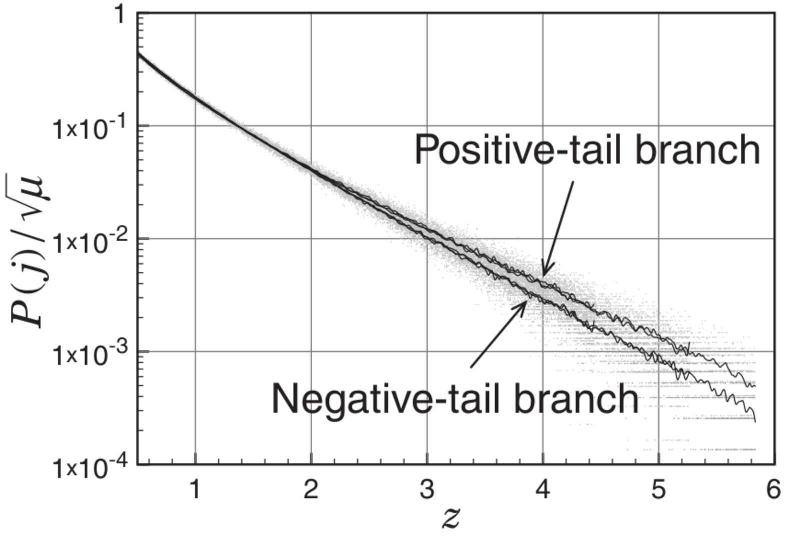
Quantum friction: 80% Overestimated

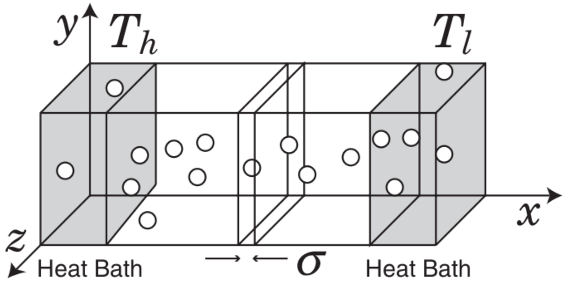
F. Intravaia et al, PRL **117**, 100412, 2016



Temperature depends on current direction

S. Yukawa et al, J. Phys. Soc. Jpn. **78**, 023002, 2009

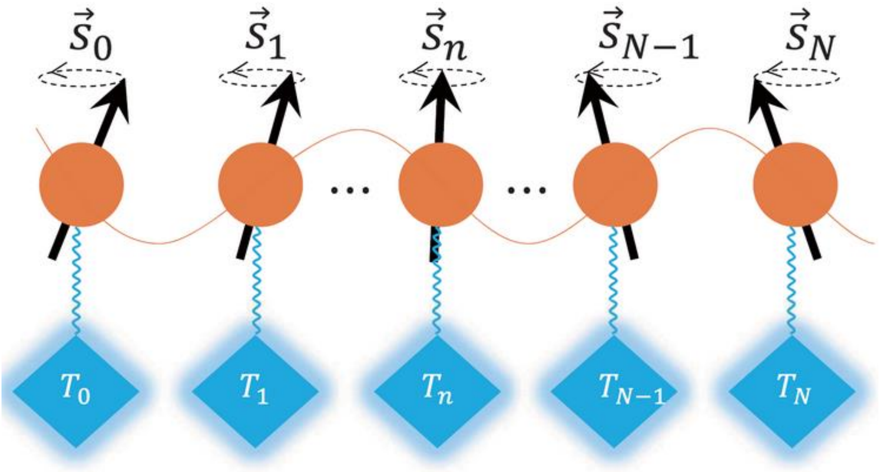




SPIN DYNAMICS IN FI

- ◆**Spin degree of freedom: Classical Heisenberg interaction**

- ◆**Eliminating the phonon (photon/electron) degree of freedom: Markov stochastic Landau-Lifshitz-Gilbert equation**
  - ❑ The bath dynamics is much faster than the spin dynamics (No works in ultrafast dynamics)



Stochastic LLG equation:

$$\dot{\vec{s}}_n = -\vec{s}_n \times \left(\vec{H}_{\text{eff}} + \vec{\eta}_n(t)\right) + \alpha \vec{s}_n \times \dot{\vec{s}}_n$$

Effective field:

$$\vec{H}_{\text{eff}} = J(\vec{s}_{n+1} + \vec{s}_{n-1}) + D s_n^z \hat{z} + \vec{H}_n$$

Fluctuation-dissipation theorem

$$\langle \eta_n^i(t)\eta_{n'}^j(t')\rangle = 2\alpha k_B T_n \delta_{ij}\delta_{n,n'}\delta(t-t')$$

LINEAR SPIN WAVE THEORY

◆**Small-angle dynamics:** $\vec{s}_n = \hat{z} + (s_n^x \hat{x} + s_n^y \hat{y})$ **with** $|s_n^{x(y)}| \ll 1$

$$\psi_n = s_n^x + i s_n^y$$

◆**Dissipative-fluctuated Schrodinger equation:**

Temperature-induced field

$$(i+\alpha)\dot{\psi}_n = J(\psi_{n+1} + \psi_{n-1} - 2\psi_n) - (H_n + 2D)\psi_n + \Theta_n$$

◆**Linear mode expansion:** $\psi_n = \sum_k P_{nk}\phi_k$

$$\Theta_n = \eta_n^x + i\eta_n^y$$

$$H_0\psi_n = J(\psi_{n+1} + \psi_{n-1} - 2\psi_n) - (H_n + 2D)\psi_n$$

$$P^{-1} H_0 P = \mathrm{diag}\{-\omega_k\}$$

$$\langle \Theta_n(t)\Theta_{n'}^*(t')\rangle = 4\alpha k_B T_n \delta_{n,n'}\delta(t-t')$$

◆**Dynamics in mode space:**

Temperature-induced field

$$(i+\alpha)\dot{\phi}_k = -\omega_k\phi_k + \Lambda_k$$

$$\Lambda_k(t) = \sum_n P_{nk}\Theta_n(t)$$

◆**Integrate from t to $t+\tau$, with $\tau_e \ll \tau \ll \tau_m$**

$$\phi_k(t+\tau) = \phi_k(t) + (i-\alpha)\omega_k\phi_k(t)\tau - (i-\alpha)\int_t^{t+\tau}\Lambda_k(t_1)dt_1$$

◆**Magnon numbers at kth mode:** $n_k = \langle\phi_k^*(t)\phi_k(t)\rangle$

◆**Magnon number dynamics:**

$$\dot{n}_k = -2\alpha\omega_k n_k + 4\alpha k_B \mathcal{T}_k$$

Mode temperature matrix:

$$\mathcal{T}_{k,k'} = \sum_n P_{nk}P_{nk'}T_n$$

◆**Initial condition:** $n_k(t=0) = 0$

$$n_k = \frac{2k_B\mathcal{T}_k}{\omega_k}(1 - e^{-2\alpha\omega t})$$

Diagonal term (Mode temperature)

$$\mathcal{T}_k = \sum_n P_{nk}P_{nk}T_n$$

◆**At steady state,** $n_k = \dfrac{2k_B\mathcal{T}_k}{\omega}$

Magnon energy v.s Mode temperature: $E_k \equiv \omega_k n_k/2 = k_B\mathcal{T}_k$

Energy repartition $E_k \equiv \omega_k n_k / 2 = k_B \mathcal{T}_k$

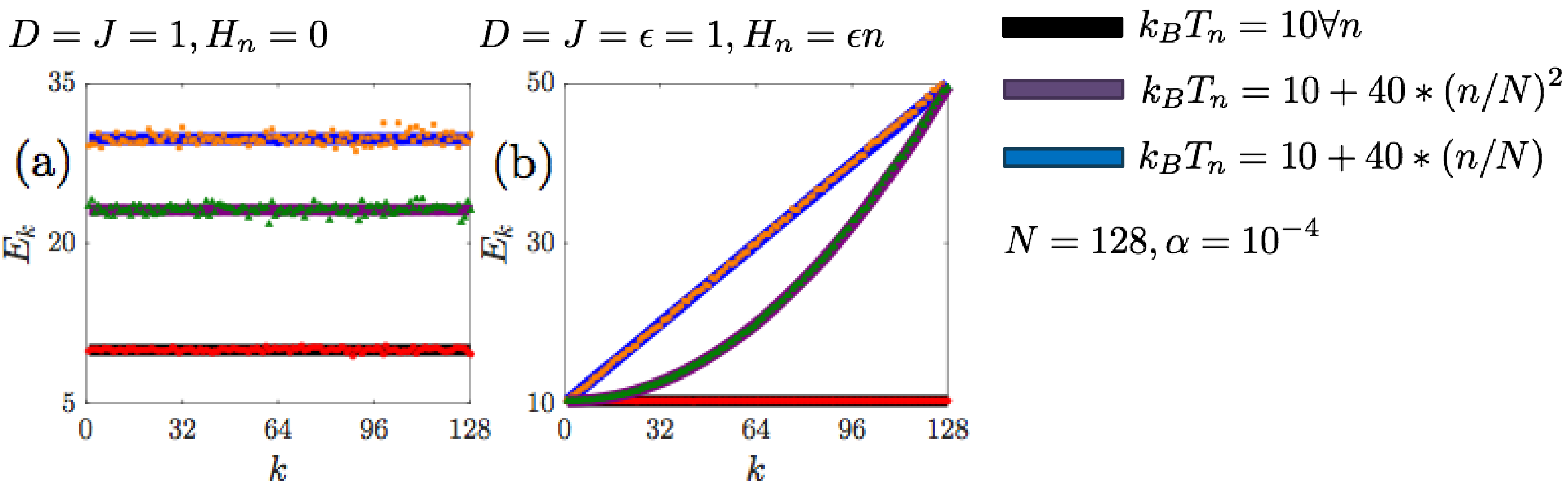
◆ **At thermal equilibrium, $\mathcal{T}_k = T_0$**

Bose-Einstein distribution

$$n_k(\omega_k) = \frac{2}{\exp(\omega_k/(k_B T_0) - 1)}$$

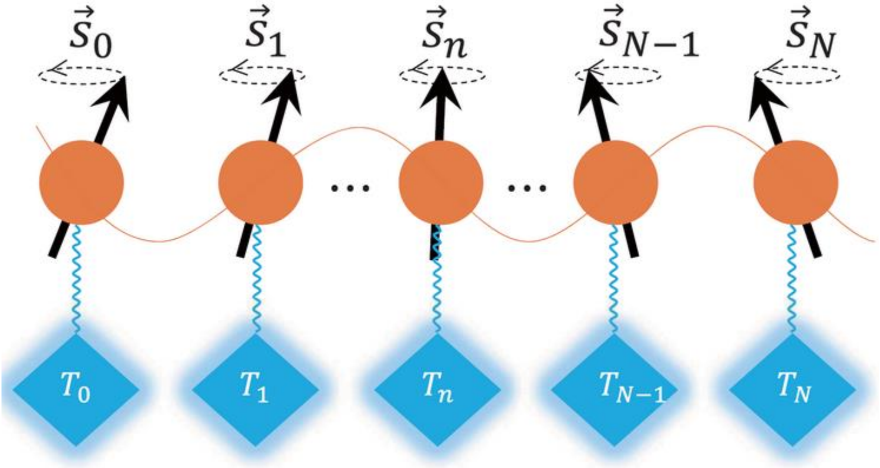
Rayleigh-Jeans distribution (high temperature limit)

$$n_k = \frac{2 k_B T_0}{\omega_k}$$



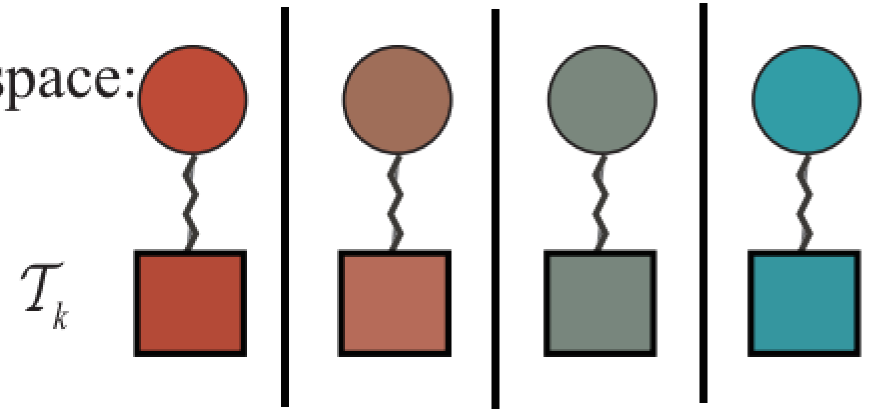
Energy repartition

Real space:



Mode temperature: $\mathcal{T}_k = \sum_n (P_{nk})^2 T_n$

Mode space:



NONLINEARITIES

Anisotropy-induced four-magnon interactions

$$(i+\alpha)\dot{\psi}_n = J(\psi_{n+1}+\psi_{n-1}-2\psi_n) - (H_n+2D)\psi_n + \Theta_n - \boxed{D|\psi_n|^2\psi_n}$$

Mode number dynamics

$$\dot{n}_k = -2\alpha\omega_k n_k + 4\alpha k_B \mathcal{T}_k - \boxed{2D\sum_{k_1k_2k_3} I_{kk_1k_2k_3}\mathrm{Re}[(\alpha-i)\langle\phi_k^*\phi_{k_1}\phi_{k_2}^*\phi_{k_3}\rangle]}$$

Overlaps between modes

$$I_{kk_1k_2k_3} = \sum_n P_{nk}P_{nk_1}P_{nk_2}P_{nk_3}$$

Resonance condition

$$k+k_2 = k_1+k_3$$

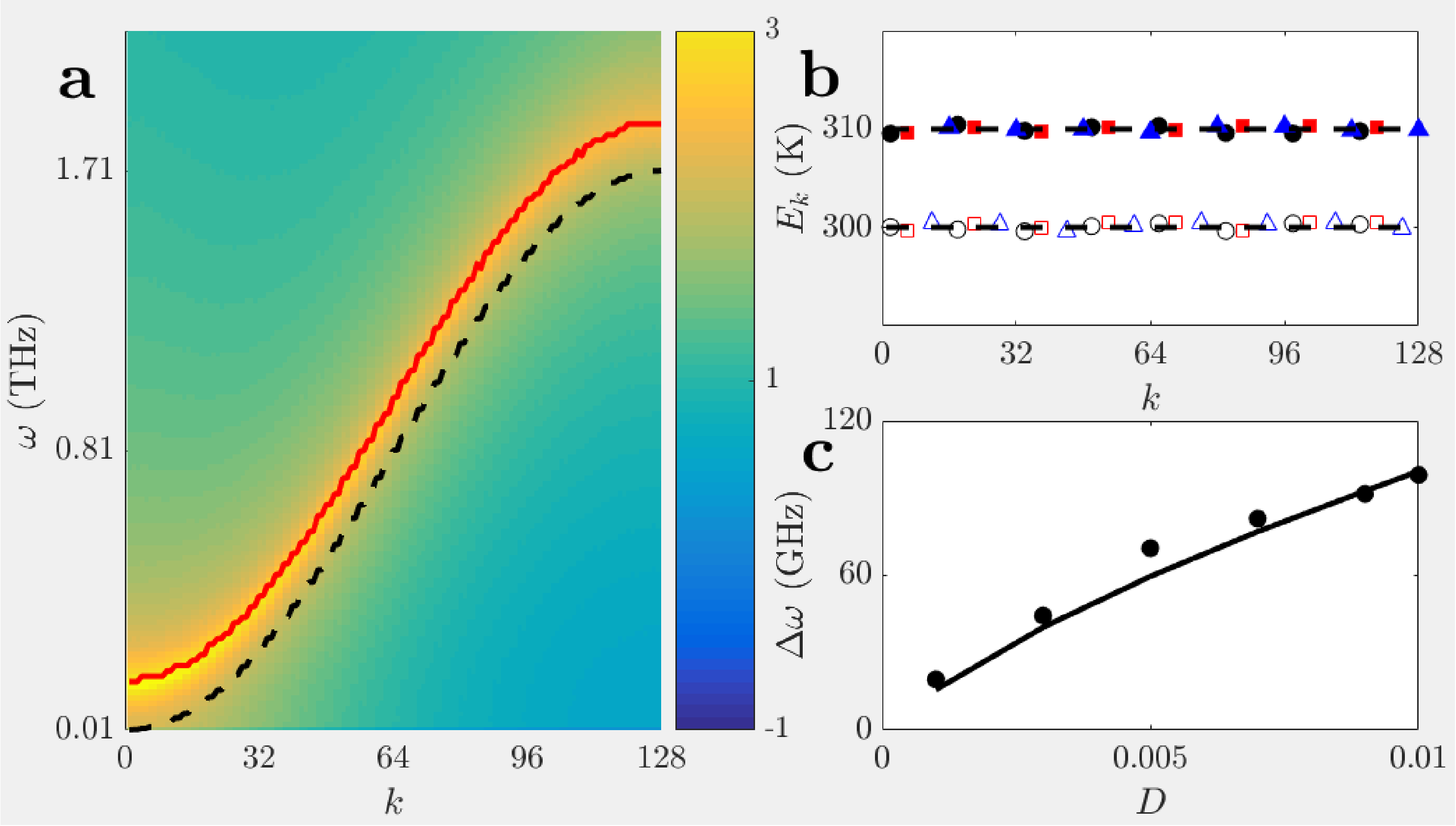
$$\omega+\omega_2 = \omega_1+\omega_3$$

$$\dot{n}_k = -2\alpha\omega_k n_k + 4\alpha k_B\mathcal{T}_k - 4D\alpha\left(\sum_{p=1}^{N} n_p/N\right)n_k$$

RENORMALIZED DISPERSION

$$\tilde{\omega}_k = \omega_k + 2D \sum_{p=1}^{N} n_p/N$$

Recover energy repartition: $n_k = 2k_B T_k/\tilde{\omega}_k$



YIG

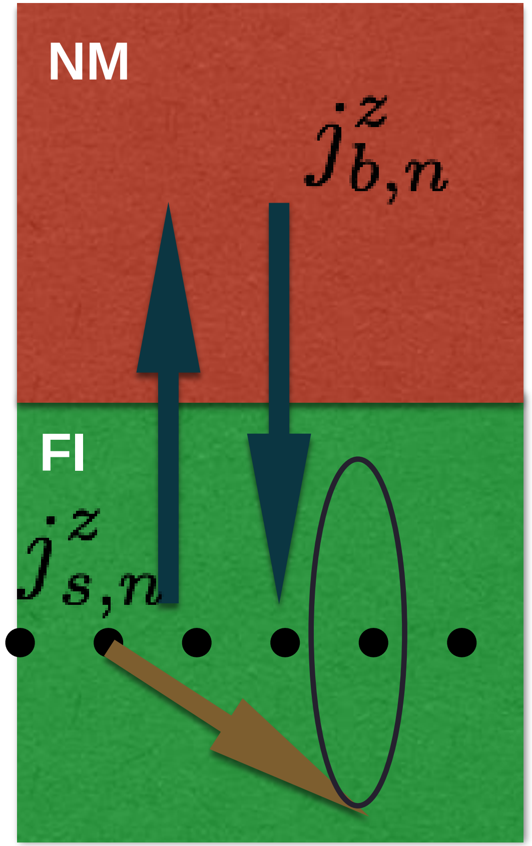
Quantity	Values
Gyromagnetic ratio γ	1.76×10^{11} rad/Ts
Gilbert damping α	10^{-4}
Saturation magnetization $4\pi S$	1.4×10^5 A/m
Exchange energy J	20 K

$$T_n = 300 \text{ K} \forall n$$

$$T_n = 300 + 20(n/N) \text{ K}$$

$$N = 128, \alpha = 10^{-4}$$

SPIN PUMPING



$$\dot{\vec{s}}_n = -\vec{s}_n \times (\vec{H}_{\text{eff}} + \vec{\eta}_n(t)) + \alpha \vec{s}_n \times \dot{\vec{s}}_n$$

Spin pumping current(dc):

$$j^z_{s,n} = \frac{g^{\uparrow\downarrow}_{\text{eff}} \hbar}{4\pi} \left\langle \vec{s}_n \times \frac{d\vec{s}_n}{dt} \right\rangle_z$$

Fluctuation current(dc):

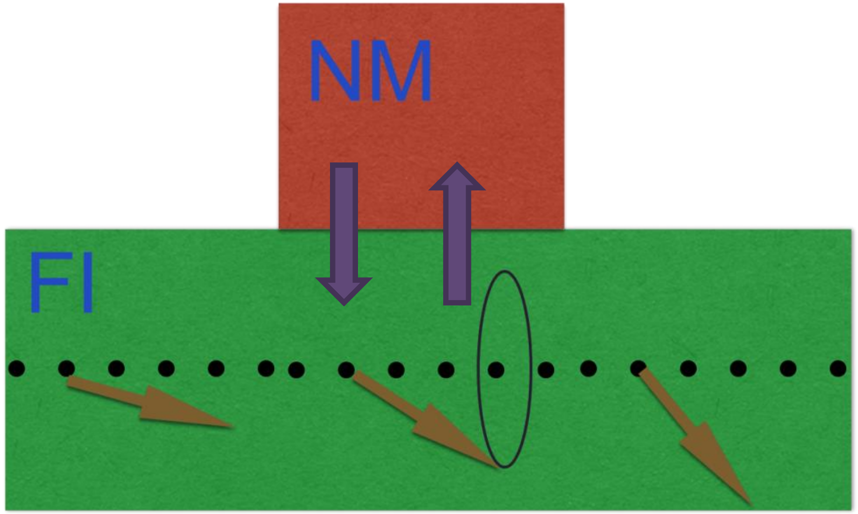
$$j^z_{b,n} = \frac{g^{\uparrow\downarrow}_{\text{eff}} \hbar}{4\pi} \left\langle \vec{s}_n \times \vec{\eta}_n \right\rangle_z$$

Inverse spin Hall effect:

$$j_n = \theta_H \frac{2e}{\hbar} j^z_{s,n}$$

Theory: Yaroslav Tserkovnyak, et al, Phys. Rev. B 66, 224403 (2002)
J. Foros, et al, Phys. Rev. Lett. 95, 016601 (2005)
Experiment: Urban R., et al, Phys. Rev. Lett. 87, 217204 (2001)
Heinrich, B., et al, J. Appl. Phys. 93, 7545 (2003)

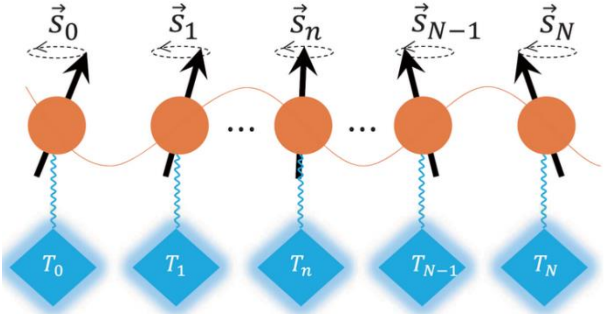
NET SPIN CURRENT



Mode temperature matrix:

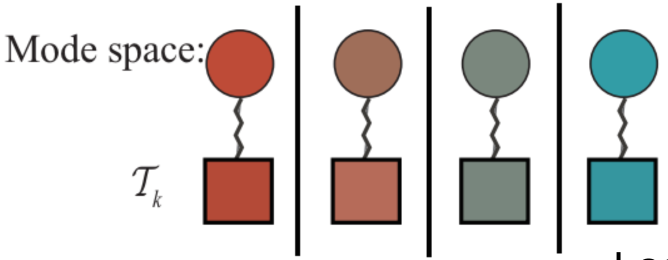
$$\mathcal{T}_{k,k'} = \sum_n P_{nk} P_{nk'} T_n$$

$$j^z_{s,n} = \frac{2\alpha^2 g_{\text{eff}}^{\uparrow\downarrow}\hbar}{\pi} \sum_{k_1 k_2} P_{nk_1} P_{nk_2} \frac{k_B(\mathcal{T}_{k_1 k_2} - T_n \delta_{k_1 k_2})\omega_{k_1}\omega_{k_2}}{\alpha^2(\omega_{k_1} + \omega_{k_2})^2 + (\omega_{k_1} - \omega_{k_2})^2}$$



Small damping (YIG)

Mode temperature: $\mathcal{T}_k = \sum_n (P_{nk})^2 T_n$



$$j^z_{s,n} = \frac{2\hbar g_{\text{eff}}^{\uparrow\downarrow}}{\pi} \sum_k (P_{nk})^2 k_B(\mathcal{T}_k - T_n)$$

Local or universal?
Depend on wave nature

Local

DISORDER

$$(i+\alpha)\dot{\psi}_n = J(\psi_{n+1}+\psi_{n-1}-2\psi_n) - (H_n+2D)\psi_n + \Theta_n$$

Inevitable material imperfection

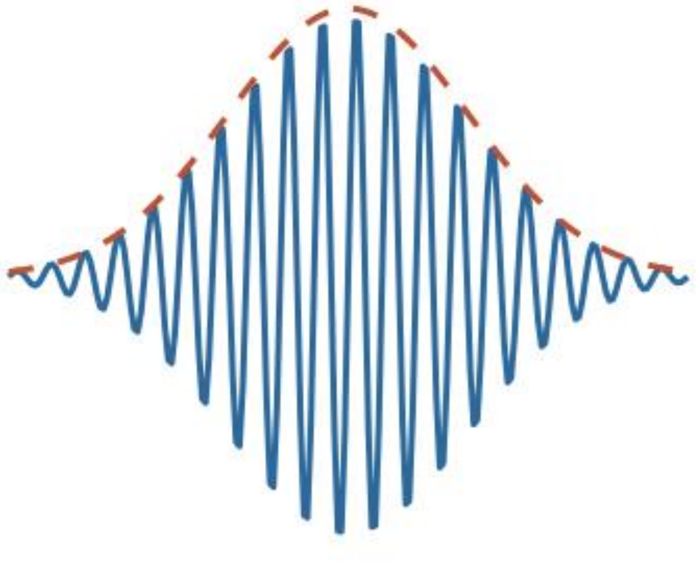
Y. Sun *et al*,
PHYSICS REVIEW LETTERS **111**, 106601, 2013

$$j^z_{s,n} = \frac{2\hbar g^{\uparrow\downarrow}_{\text{eff}}}{\pi}\sum_k (P_{nk})^2 k_B(\mathcal{T}_k - T_n)$$

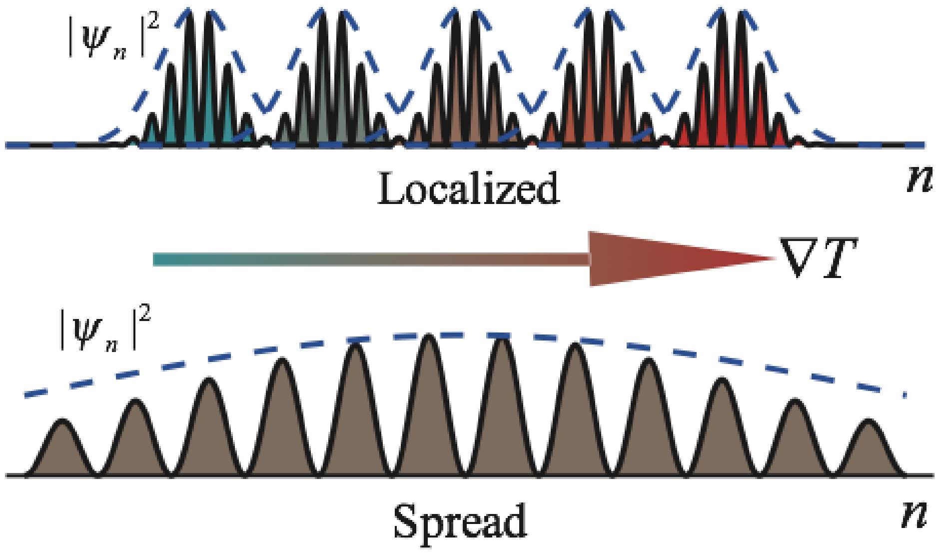
Modeling disorder

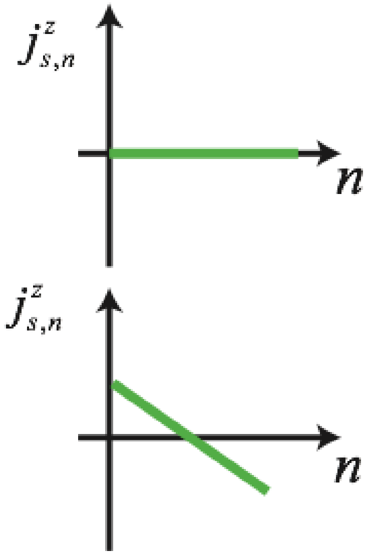
$$H_n + 2D \in [-W/2, W/2]$$

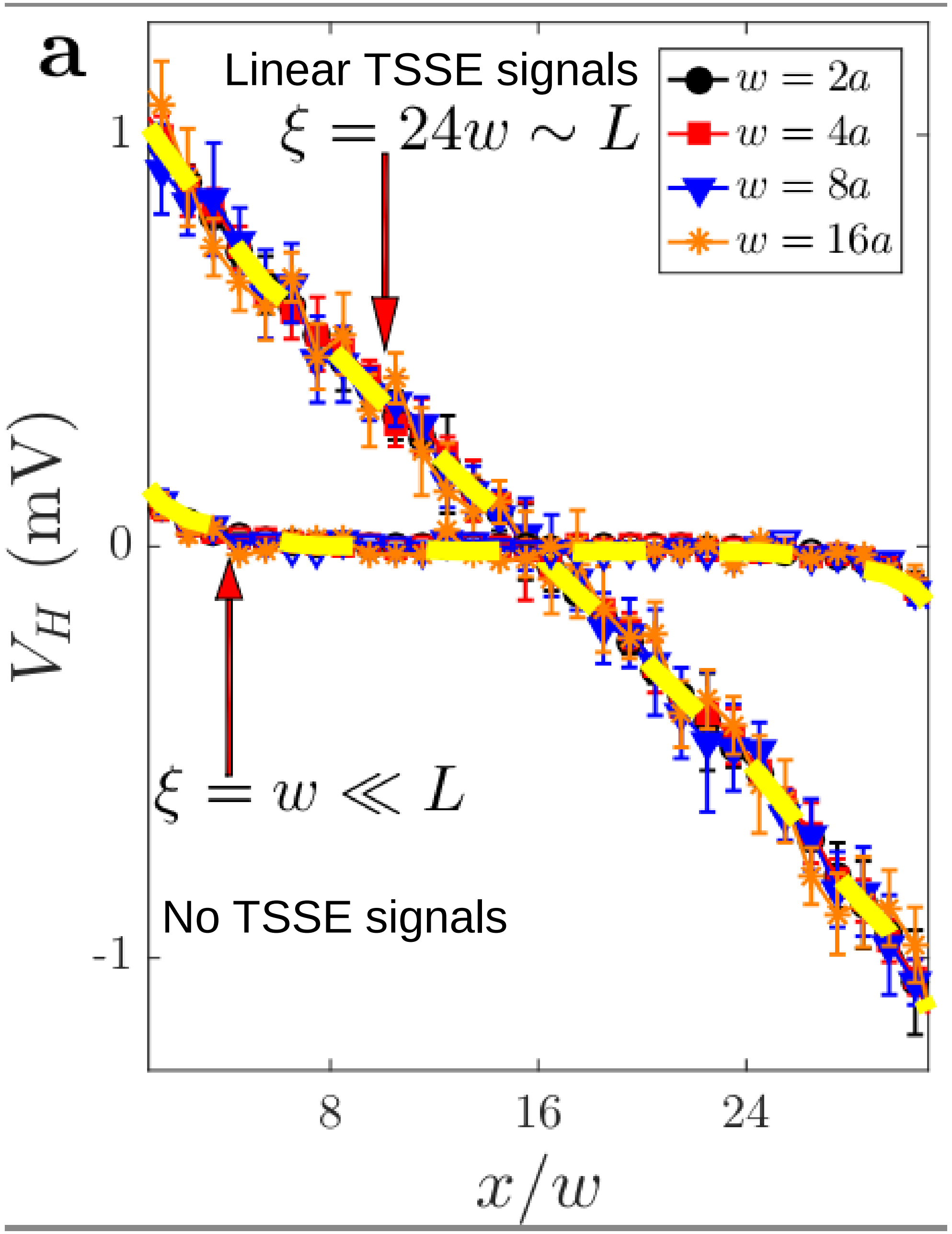
$$\mathcal{T}_k = \sum_n (P_{nk})^2 T_n = \begin{cases} \bar{T} & \text{Extended} \\ T_n & \text{Localized} \end{cases}$$



$$\xi \simeq 96J^2/W^2$$

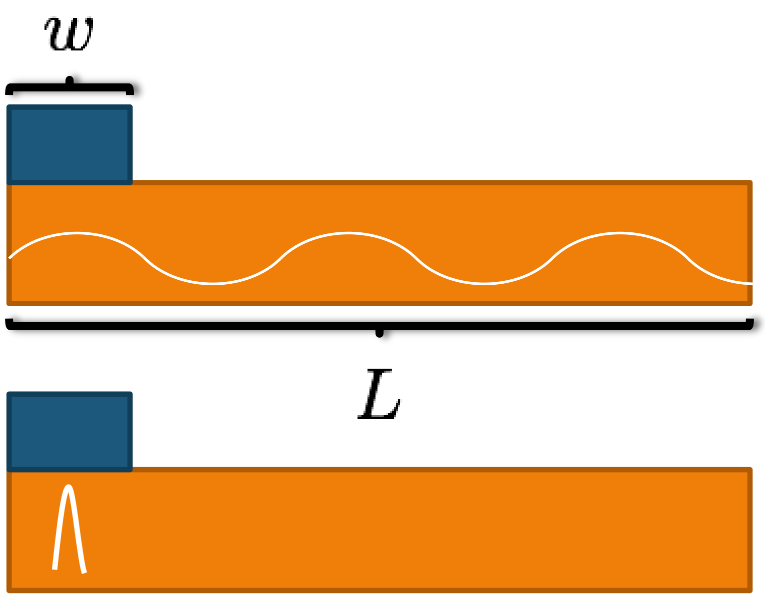






No extended spin wave
in disordered 1D systems

$\xi \simeq 96J^2/W^2$



$$\text{Fix } L = 32w, \text{scaling system by changing } w$$

	YIG	
Quantity	Values	References
Gyromagnetic ratio γ	1.76×10^{11} rad/Ts	
Gilbert damping α	10^{-4}	[42]
Saturation magnetization $4\pi S$	1.4×10^5 A/m	[42]
Exchange energy J	20 K	[43, 44]
Nonlinearity ν	0.02 K	[26]
Mixing conductance $g_{\mathrm{eff}}^{\uparrow\downarrow}/A$	$3 \times 10^{12}\mathrm{cm}^{-2}$	[42]
	Pt contact	
Quantity	Values	References
Hall angle θ_H	0.00037	[45]
Resistance ρ	1×10^{-6} Ωm	[1]
Geometry $l \times w \times h$	4 mm $\times$ 0.1 mm $\times$ 15 nm	[1]

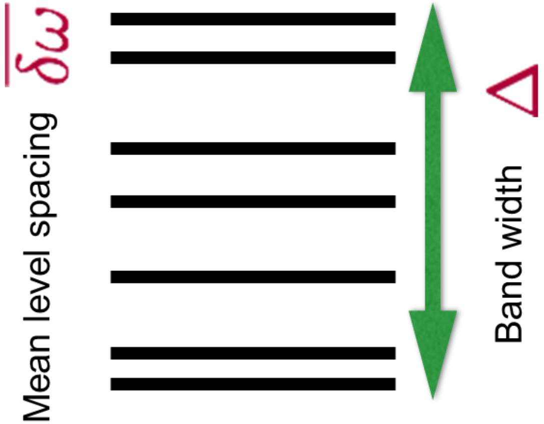
TABLE II. Parameters for the YIG|Pt system.

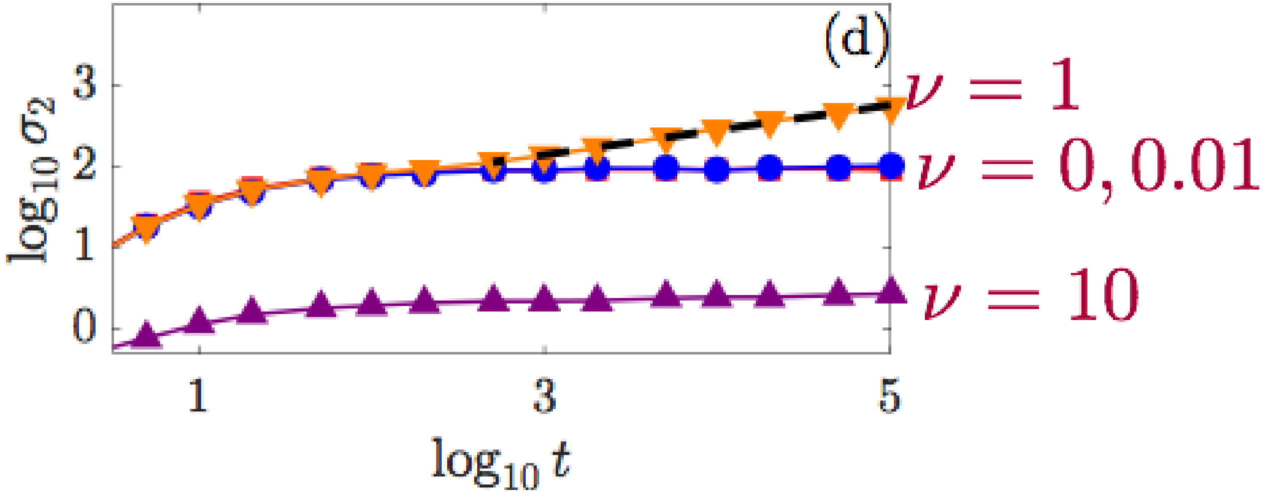
NONLINEAR EFFECT

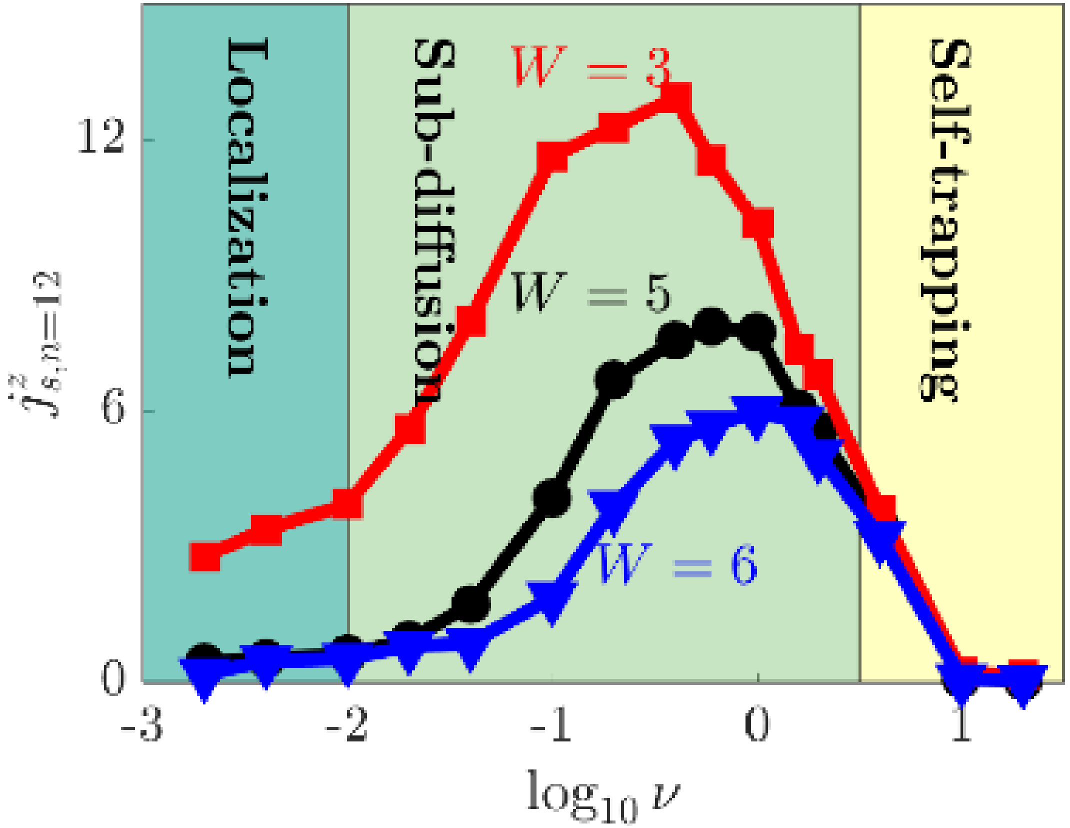
Nonlinearity delocalizes localized spin waves

A. S. Pikovsky *et al*,
PHYSICS REVIEW LETTERS **100**, 094101, 2008

Localization	Sub-diffuison	Self-trapping
	Delocalization!!	
$< \overline{\delta\omega}$	$\overline{\delta\omega} < \cdots < \Delta$	$> \Delta$







CONCLUSION

- We propose the energy of magnon follows energy repartition.
- Based on energy repartition, we can define a mode temperature for magnon in NESS.
- Net spin current is related to the difference between mode temperature and local temperature.
- Disorders localize spin waves, and suppress the SSE.
- Moderate nonlinearity delocalize spin waves, and enhance the SSE.