Speedup of the Quantum Adiabatic Algorithm by Topological Cancellation

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Quantum Adiabatic Algorithm (QAA)

- Unique among possible quantum algorithms because it is “general purpose” – most optimization problems can formulated to use it (also QAA is technically universal)
- Devices meant to implement it have been constructed (D-Wave), so it may be the first algorithm to run on an actual quantum computer
- Still not clear whether there are any problems for which the QAA is more efficient than various classical optimization algorithms

Outline

- A Very Short Introduction to the Quantum Adiabatic Algorithm (QAA)
- Topology of the QAA
- Topological Cancellation
- Improved QAA: Single-Spin Approximation
- Improved QAA: Cluster Approximation
- Conclusions

Basics of the QAA

A quantum computer evolves according to the time-dependent Hamiltonian

$$H_{qaa} = \left(1 - \frac{t}{t_D}\right) H_i + \frac{t}{t_D} H_f$$

Here H_i is any sufficiently simple Hamiltonian, e.g., : $H_i = h_0 \sum_{n=1}^{L} \sigma_x^{(n)}$

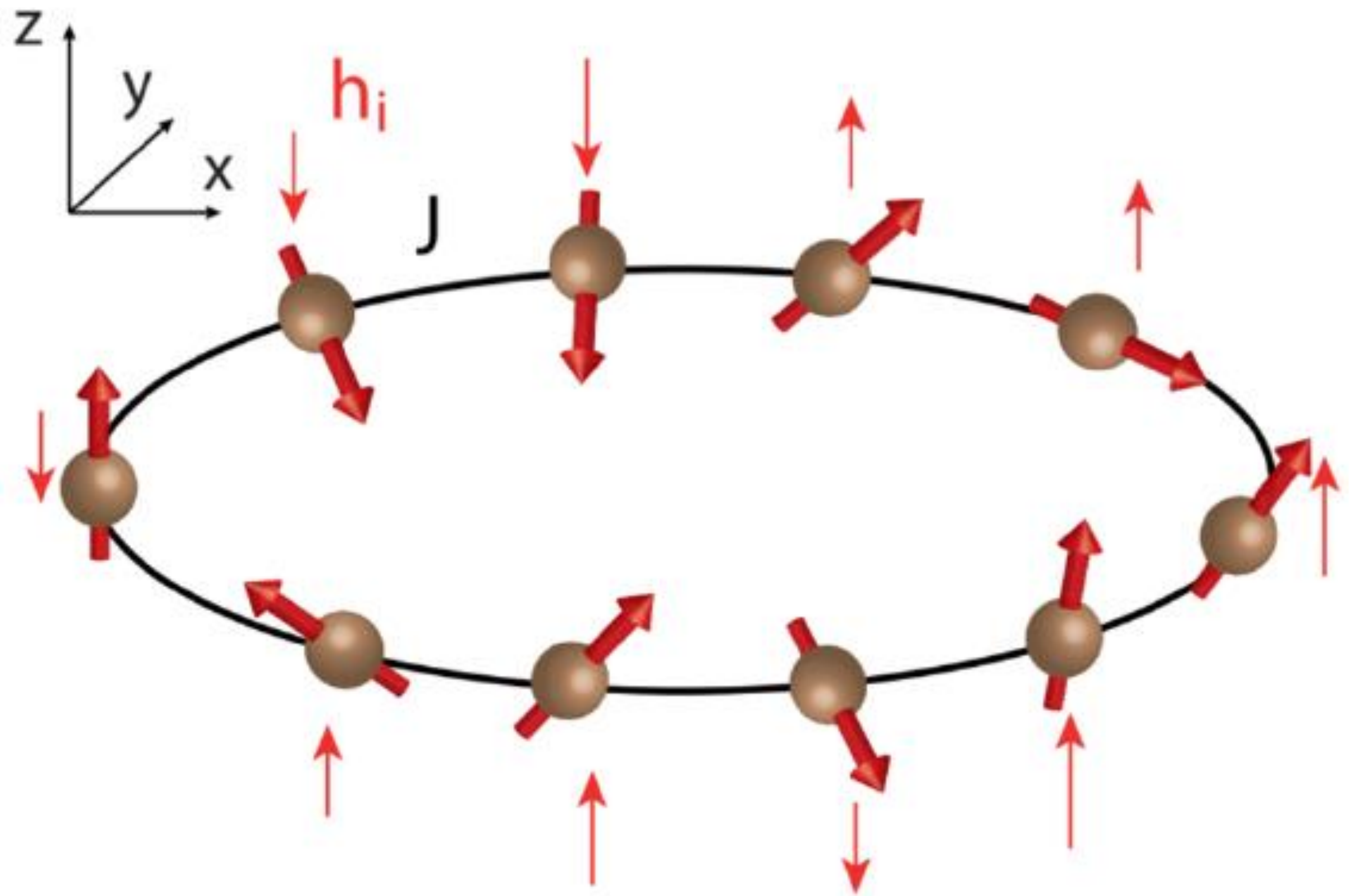
ground state $|\Psi_i\rangle = 2^{-L/2} \left(|0\rangle - |1\rangle\right)_1 \left(|0\rangle - |1\rangle\right)_2 \cdots \left(|0\rangle - |1\rangle\right)_L$

and H_f is a complicated Hamiltonian whose ground state encodes the solution to the optimization problem, e.g.,

$$H_f = \sum_{n=1}^{L} h_n \sigma_z^{(n)} + J \sum_{n=1}^{L} \sigma_z^{(n)} \sigma_z^{(n+1)}, h_n \in [-1, 1]$$

Specification of $\{h_n\}$ determines an instance (or a realization) of the optimization problem.

Random-field Ising Model with t-dependent transverse field



Variable Number L of spins on the Ring

Adiabaticity

- If t_D is large enough, then the adiabatic theorem tells that $|\Psi_i\rangle$ will evolve into $|\Psi_f\rangle$ at $t = t_D$, and that $|\Psi_f\rangle$ is the ground state of H_f, thus solving the optimization problem.

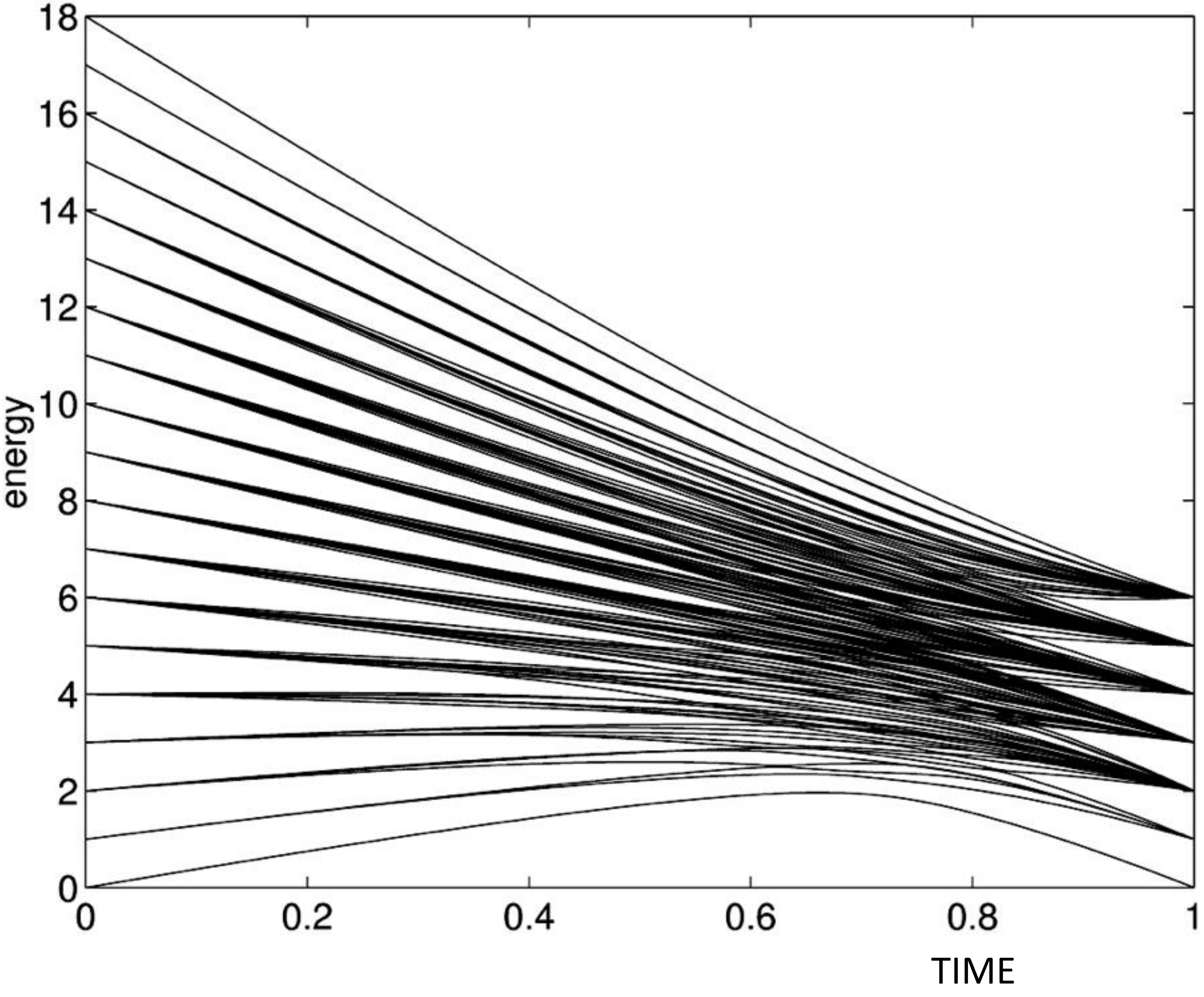
How large is large enough for t_D? Simple arguments (see, e.g., Messiah, 1961) say that if we define the minimum gap as:

$$\Delta_{\min} = \min_{0 \le t \le t_D} \left[E_1(t) - E_0(t) \right]$$

Then $$t_D >> C \left\| (dH/dt)^2 \right\| \; / \; \Delta_{\min}^2$$

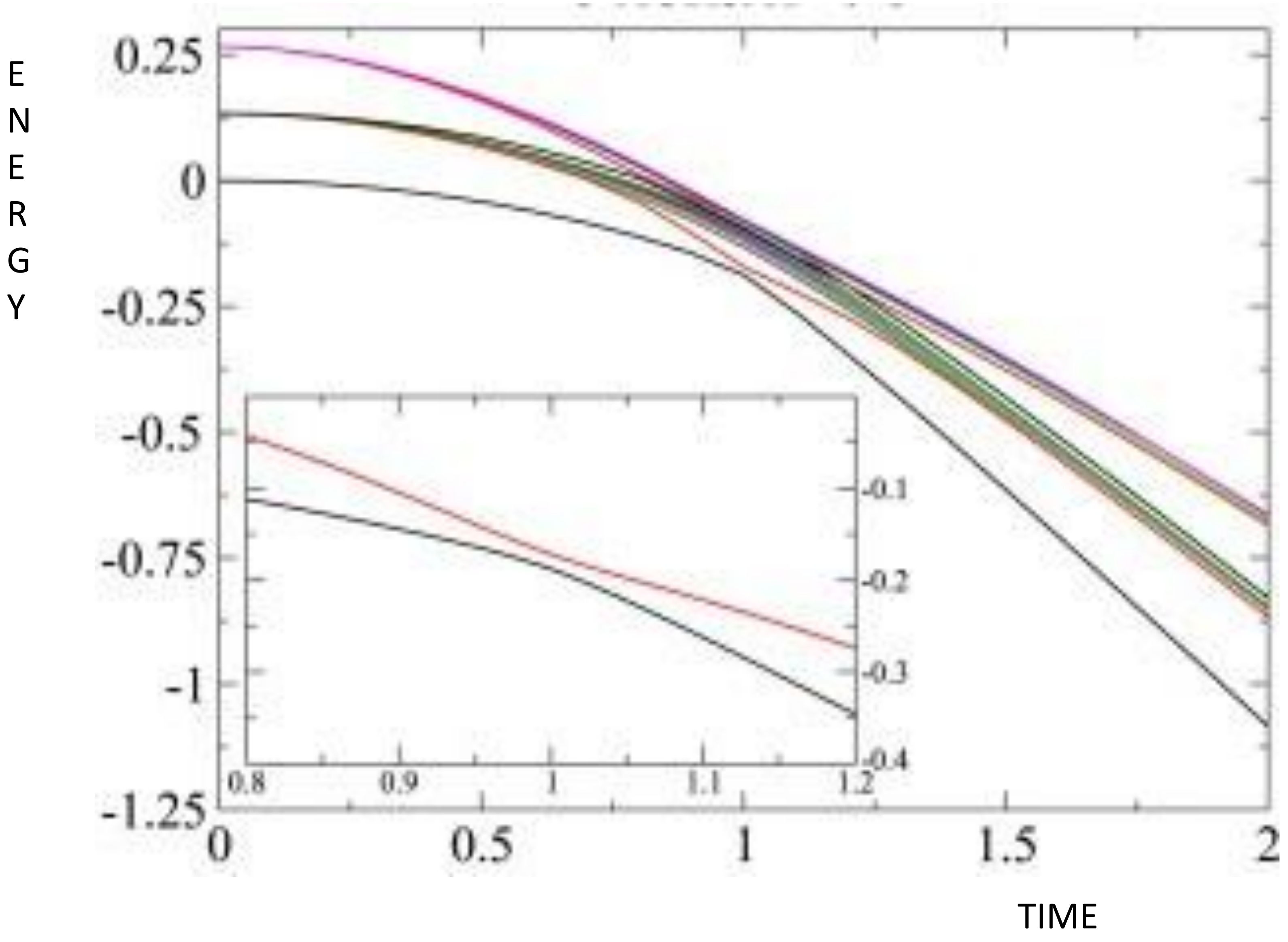
If $\Delta_{min} = O(L^{-n})$ the algorithm is efficient but if $\Delta_{min} = O(\exp(-L))$ then the algorithm is inefficient

GOOD!



AM Childs talk

BAD!

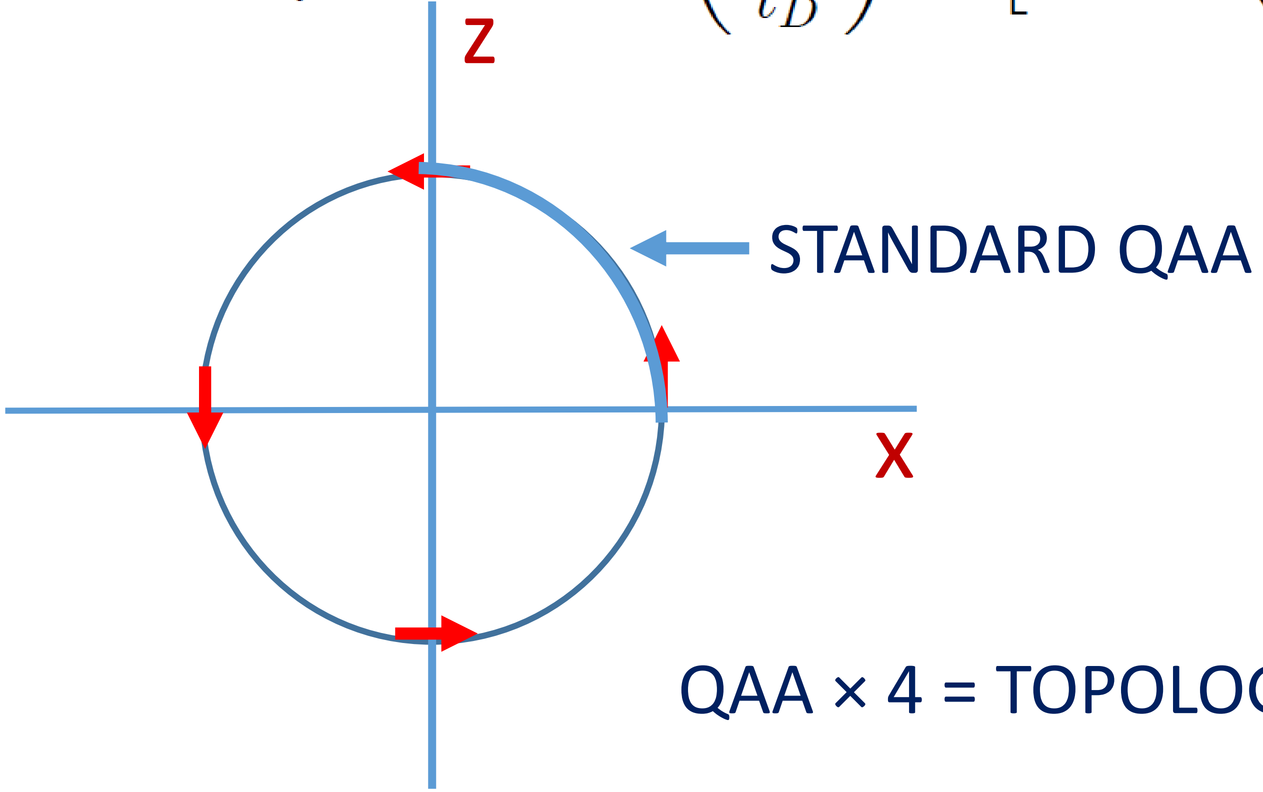


Bapst et al., Phys. Rep. 523, 127 (2013)

Topological Picture of the QAA

- Consider an *expanded* time evolution: $H_{topo}(t) = \cos\left(\frac{2\pi t}{t_D}\right) H_i + \sin\left(\frac{2\pi t}{t_D}\right) H_i$
- And an effective mean field on the *n*th spin:

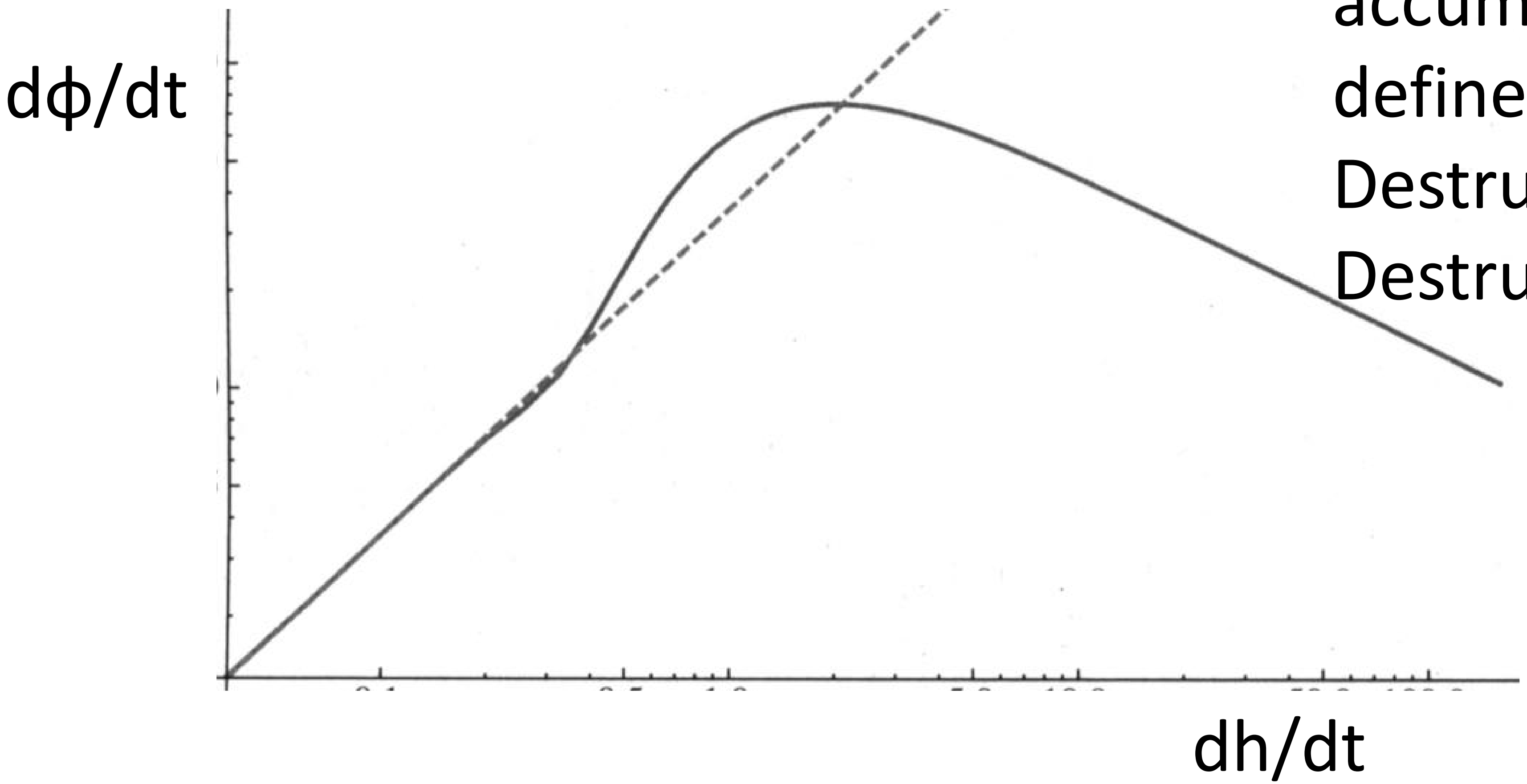
$$h_{mf}^{(n)}(t) = h_0 \cos\left(\frac{2\pi t}{t_D}\right)\hat{x} + \left[h_n + J\left(\sigma_z^{(n+1)}(t) + \sigma_z^{(n-1)}(t)\right)\hat{z}\right]\sin\left(\frac{2\pi t}{t_D}\right)$$



Perfect adiabatic evolution gives M_y=0 and Berry phase = π. *Deviations from π indicate breakdown of the algorithm.*

QAA × 4 = TOPOLOGICAL QAA

Breakdown of Adiabatic Berry Phase Accumulation Spin ½ Particle in a t-dependent B-field.



Sharp Breakdown of adiabatic accumulation at a fairly well-defined driving speed. Destruction of Topology = Destruction of QAA!

Gritsev and Polkovnikov, PNAS **109**, 6457 (2011)

Topological Cancellation

Define a time-dependent Hamiltonian $H_0(t)$ with instantaneous eigenstates $|n(t)>$:

$$H_0(t)\;|n(t)\rangle = E_n(t)\;|n(t)\rangle$$

Then there will be transitions between the $|n(t)>$. But if we define $H = H_0(t) + H_1(t)$, with

$$H_1(t) = i\sum_{m\neq n}\frac{|m\rangle\langle m|\partial_t H_0|n\rangle\langle n|}{(E_n - E_m)}$$

BERRY CONNECTION TERM
ALSO “COUNTER-DIABATIC” TERM !

Then if the initial state is $|n(t)>$, the final state is also $|n(t)>$.
For a single spin these equations reduce to

$$H_0(t) = -\vec{B}(t)\cdot\vec{\sigma}/2$$ and $$H_1(t) = \frac{\vec{B}\times\partial_t\vec{B}}{2B^2}\cdot\vec{\sigma}$$

MV Berry, J. Phys. A 42, 365303 (2009)

Application to the QAA

We propose to add the topological term to the Hamiltonian that will suppress the unwanted transitions:

$$\begin{aligned} H_{total}(t) &= f_i\left(\frac{t}{t_D}\right)H_i + f_f\left(\frac{t}{t_D}\right)H_f + H_s(t) \\ f_i(0) &= f_f(1) = 1;\ f_i(1) = f_f(0) = H_s(0) = H_s(1) = 0 \end{aligned}$$

H_s must vanish at both endpoints: at t=0 because it is not easy to diagonalize, at $t=t_D$ because it must not change the optimization problem.
H_s involves the instantaneous eigenstates, which we do not know how to calculate efficiently: must approximate!

Single-particle Approximation

Now we choose $f_i\left(t\right) = \cos^2\left(\frac{\pi t}{2t_D}\right);\ f_f\left(t\right) = \sin^2\left(\frac{\pi t}{2t_D}\right)$

and $$H_s\left(t\right) = -\sum_n \frac{h_0 h_n \sin\left(\frac{\pi t}{2t_D}\right)\cos\left(\frac{\pi t}{2t_D}\right)}{2t_D\left[h_0^2\sin^4\left(\frac{\pi t}{2t_D}\right) + h_n^2\cos^4\left(\frac{\pi t}{2t_D}\right)\right]}\sigma_y^{(n)}$$

as our approximation to H_s.

It's necessary that the turn on and turn off are quadratic – otherwise it is impossible to get $H_s(t=0) = H_s(t=t_D) = 0$

Non-interacting Spins: J=0

$$P_0 = |\langle 0|\psi(t = t_D\rangle|^2$$

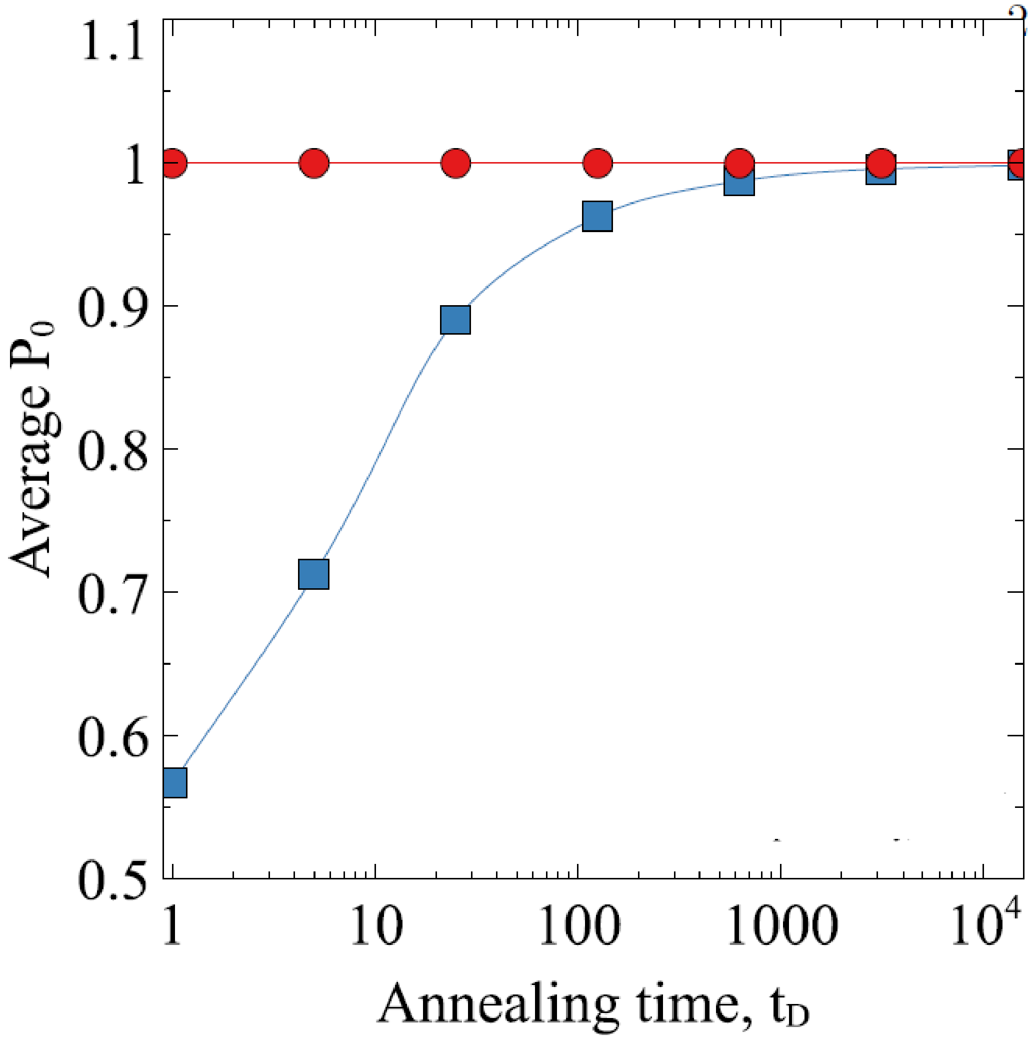
is the success probability.

Random-field Ising model with L spins on a ring.

Red points have steering.
Blue points have no steering.

10^4 realizations of the disorder.

L=1, h_0 = 10



Weakly Interacting Spins: (J=0.1)

$$P_n = |\langle n|\psi(t=t_D\rangle|^2$$

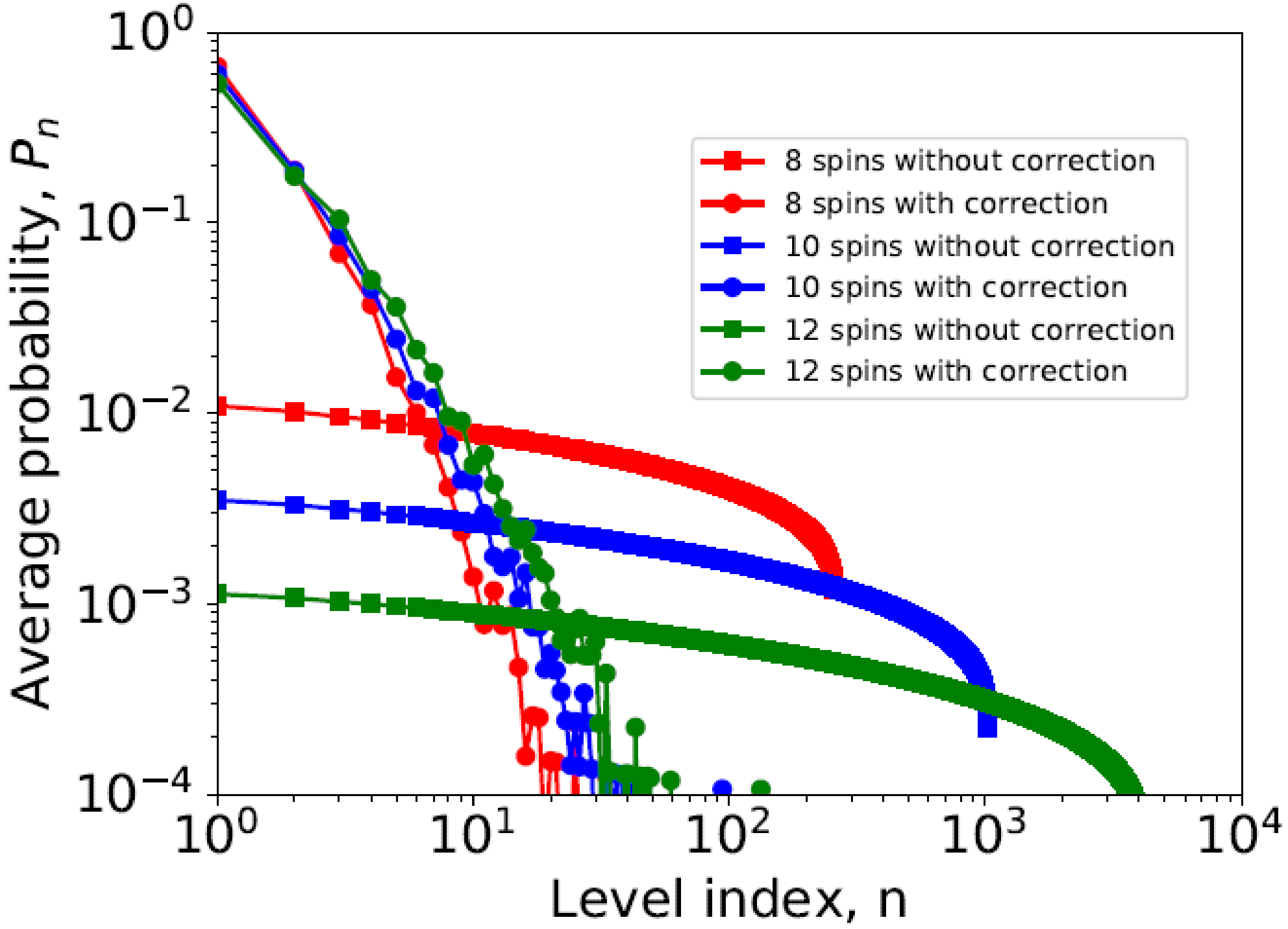
is the probability of ending in the state |n >.

Round points are with steering

Square points are without

10^4 realizations of the disorder

L = 8, 10, 12; t_D = 1 (short!)



Interacting Spins: (all J)

$$P_0 = |\langle 0|\psi(t = t_D\rangle|^2$$

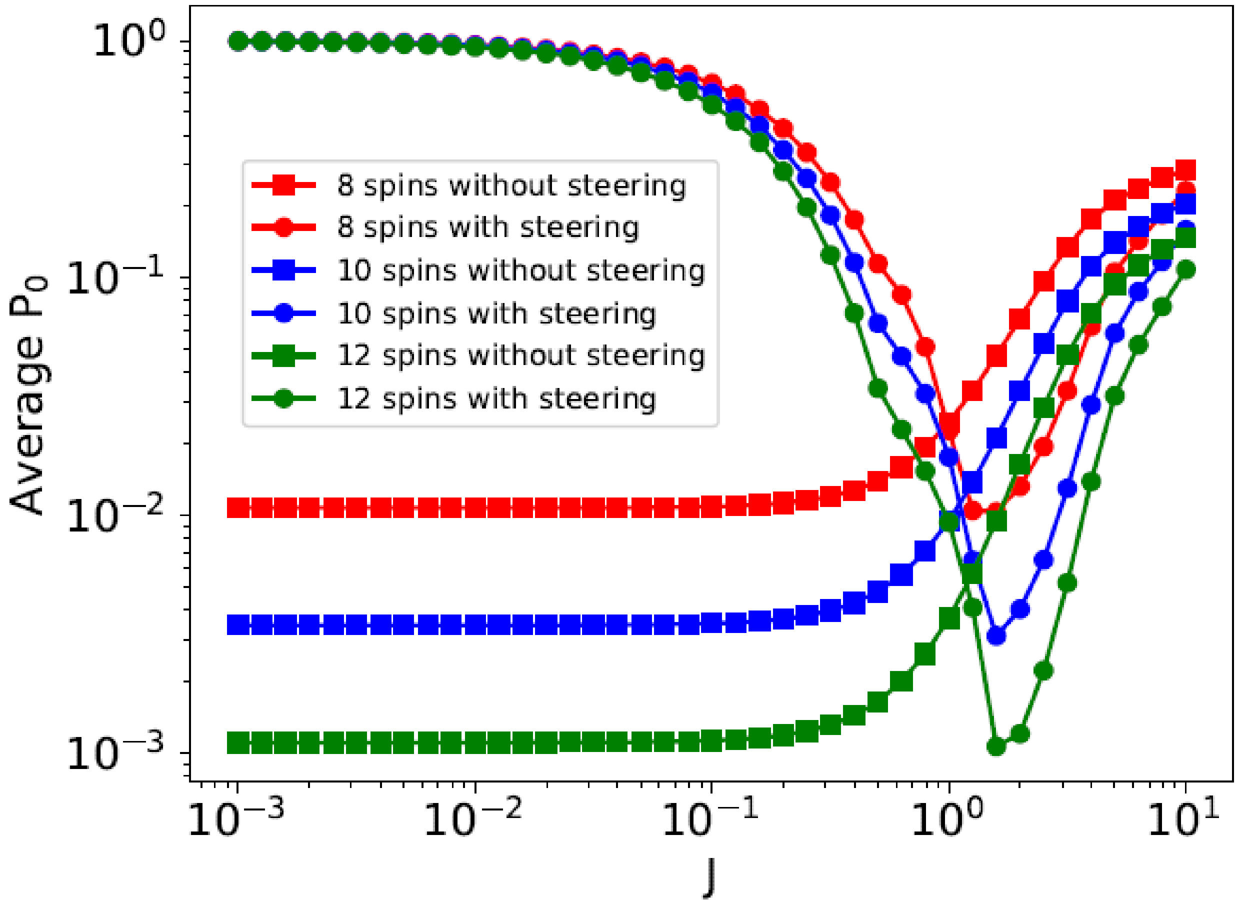
is the probability of ending in the state |0 >.

Round points are with steering

Square points are without

10^4 realizations of the disorder

L = 8, 10, 12; t_D = 1 (short!)



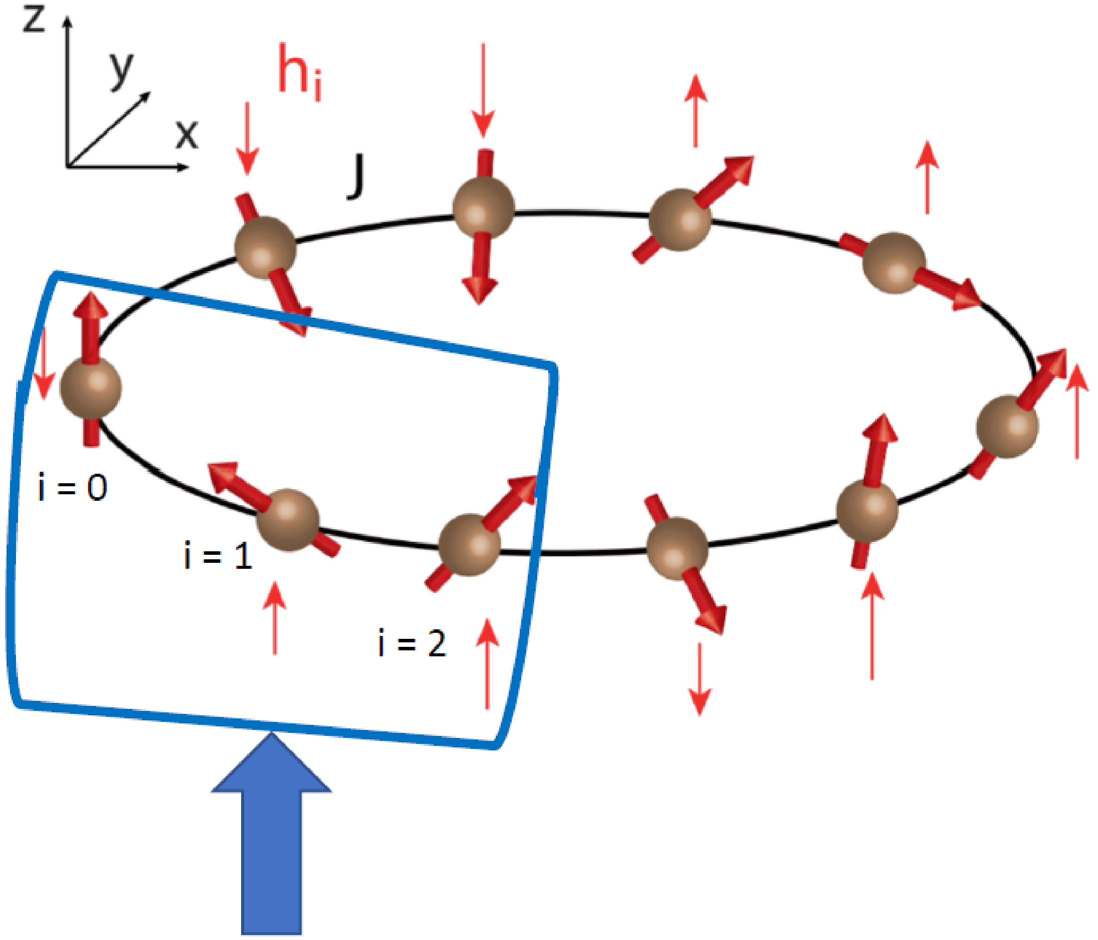
Single-spin Approximation: Remarks

- The algorithm is straightforward to implement.
- The calculation of the steering Hamiltonian is efficient.
- The speedups are very considerable for small J, say $J < 0.5$.
- At intermediate coupling $1 < J < 10$, single-spin steering is actually detrimental (but see below).
- At very large J, single-spin steering makes no difference.
- Can we improve?

Cluster Approximation

- We search for the spin with smallest h_n.
- The idea is that this spin is determined by the interaction and neighboring spins: it “feels” the interaction most strongly.
- This spin is chosen as the center of a cluster for which Hs is computed numerically using the exact many-spin Berry formula.
- The other spins are treated as before.
- This is the first step in a cluster expansion for the method.

Cluster approach



- For a spin with weak z-field, dynamics is governed by neighbors.
- Number spins so that h_1 is the smallest z-field.
- Apply exact Berry steering to spins numbered as 0, 1 and 2.
- To the rest of the spins, apply 1-spin steering.

Interacting Spins: Cluster Approximation

Percentage of realizations with

$P_0 = |\langle 0|\psi(t = t_D)\rangle|^2$ > 0.99 .

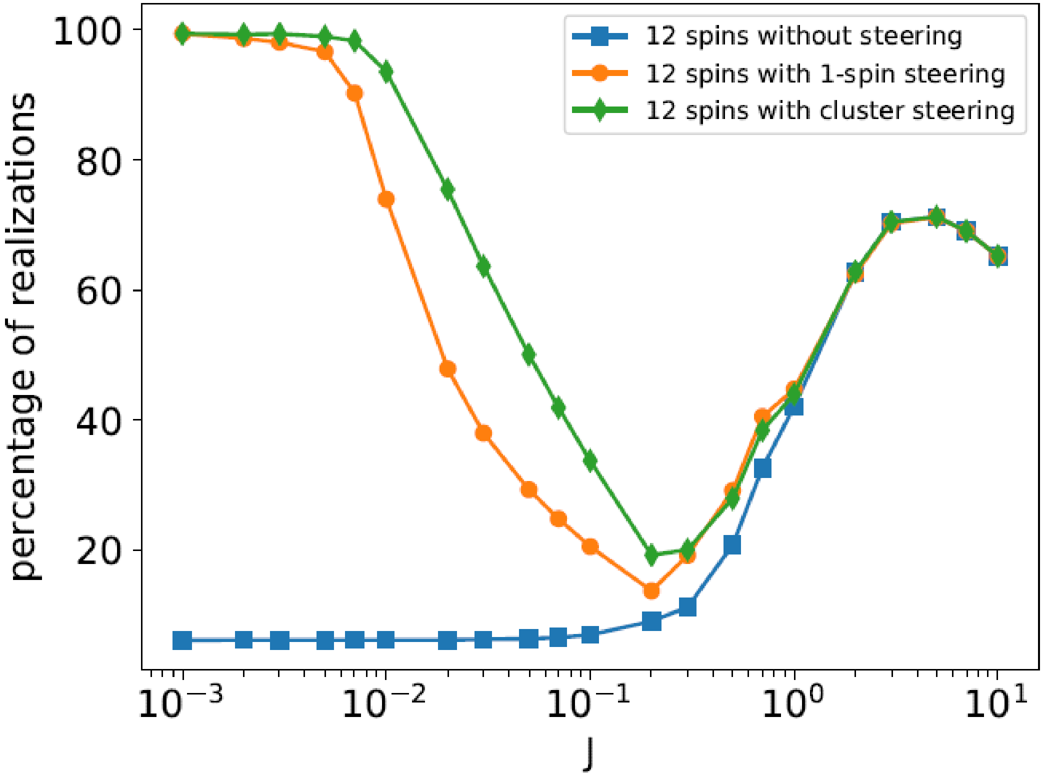
Diamond points are with cluster steering

Round points are with single-spin steering

Square points are without steering

10^4 realizations of the disorder

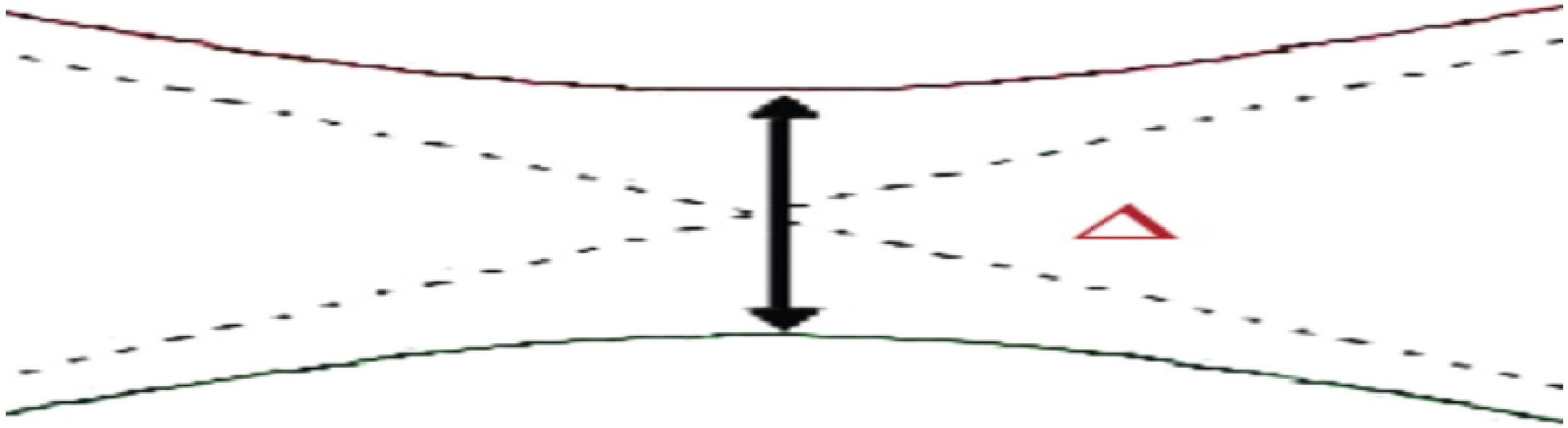
L = 12; t_D = 128



Conclusions, Discussion

- We add a topological cancellation term to the QAA Hamiltonian that suppresses single-spin transitions.
- The single-spin method shows significant improvements on the standard QAA for random spin systems with weak to moderate interaction strengths.
- The simplest cluster expansion of the cancellation term also gives a speedup in the range of moderate interactions – this suggests that systematic improvement is possible.
- H_i and H_f are stoquastic, but H_s is not.
- We have not investigated the scaling behavior of the method – its local character so far would suggest a constant speedup, but the method in general need not be local.

Landau-Zener-Majorana tunneling

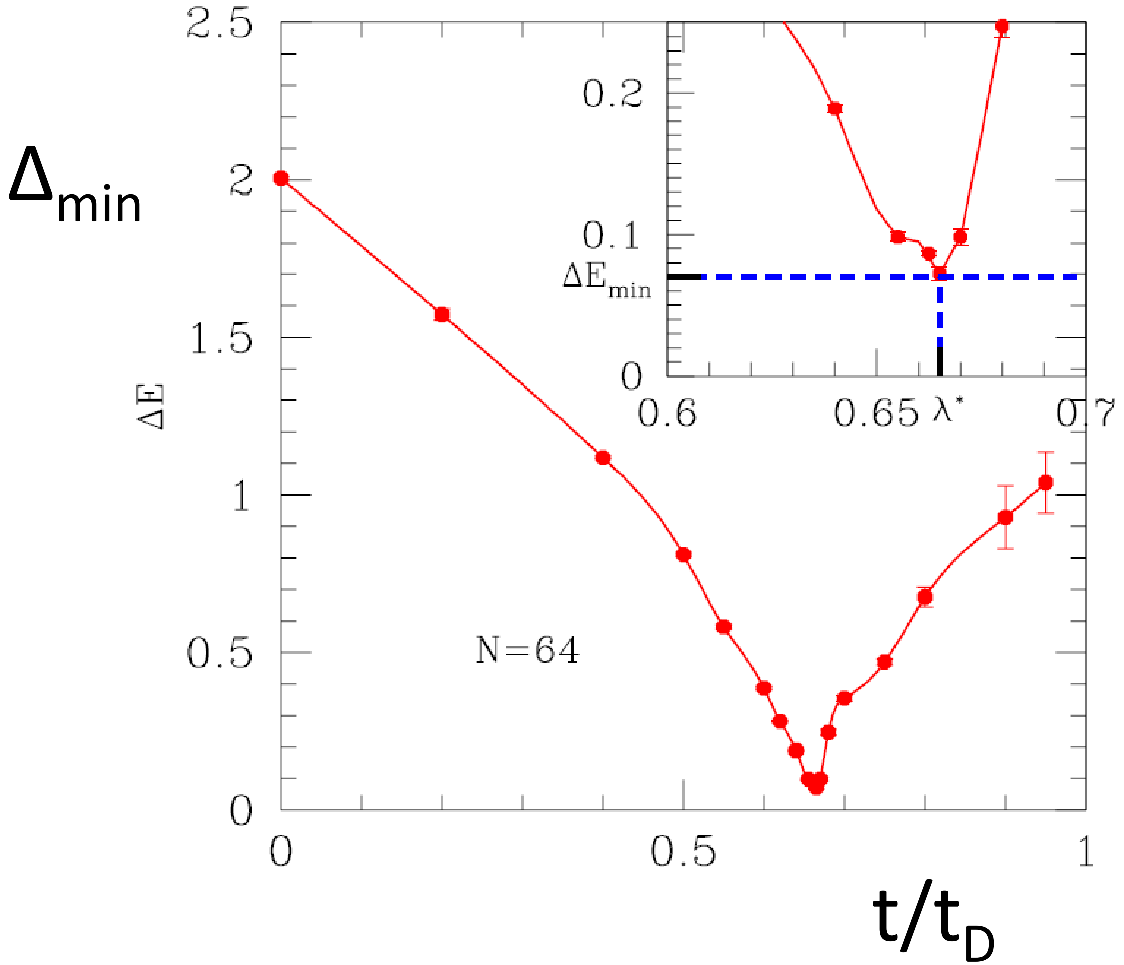


$$H_{LZ} = -\frac{vt}{2}\sigma_z - \frac{\Delta}{2}\sigma_x$$

Δ is minimum gap.

$$P_{transition} = e^{-\frac{\pi\Delta^2}{2v}}$$

LZM Tunneling *is* the Problem



$$t_D >> \hbar \frac{|H_{10}(t = t_{\min})|}{\Delta_{\min}^2}$$

Seems to be a very accurate criterion in the problems known to date, indicating that individual avoided Level crossings are the main obstacle to the efficiency of the QAA.

Young, Knysh, Smelyanskiy, PRL 101, 170503 (2008)