KITS, 2017

Spinon walk in quantum spin ice

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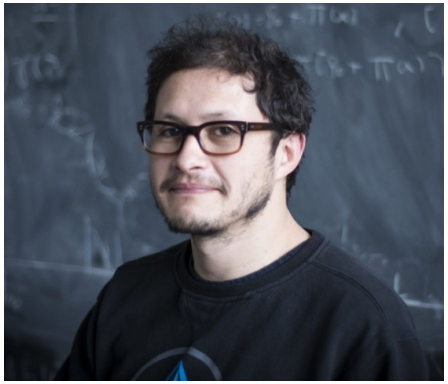
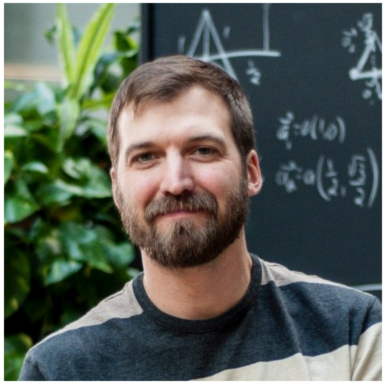
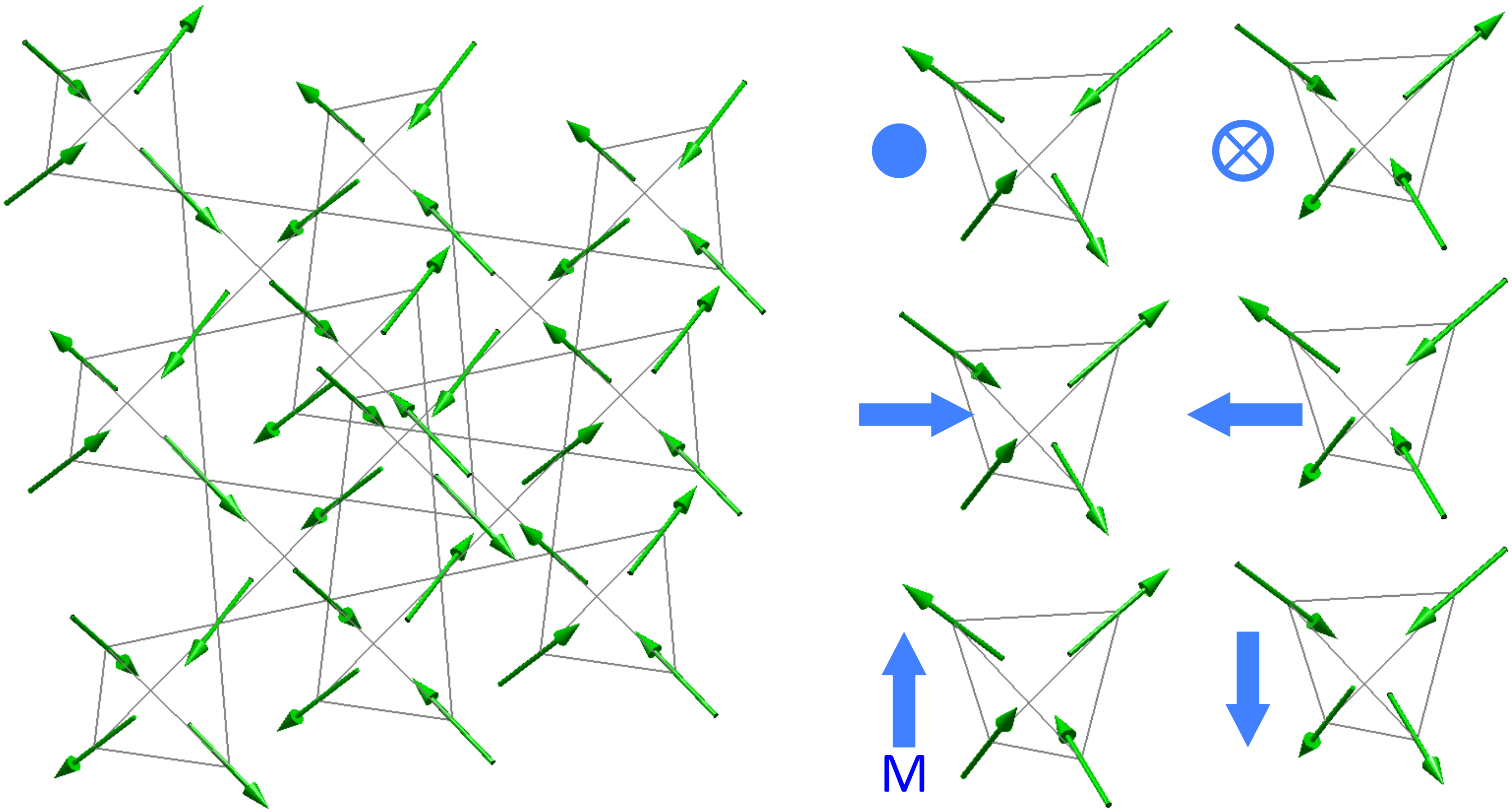


PI -> D wave



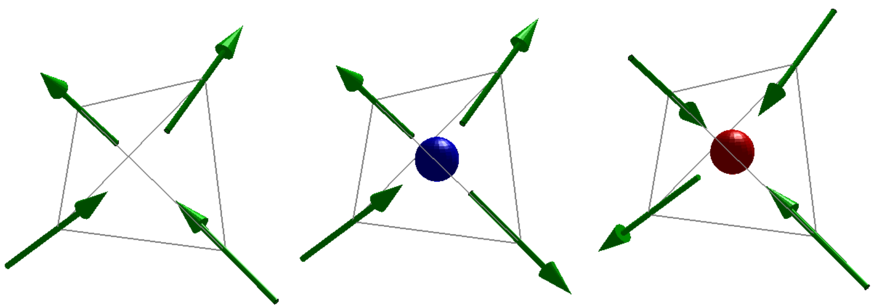
UW,PI

Spin ice



Freedom of poles

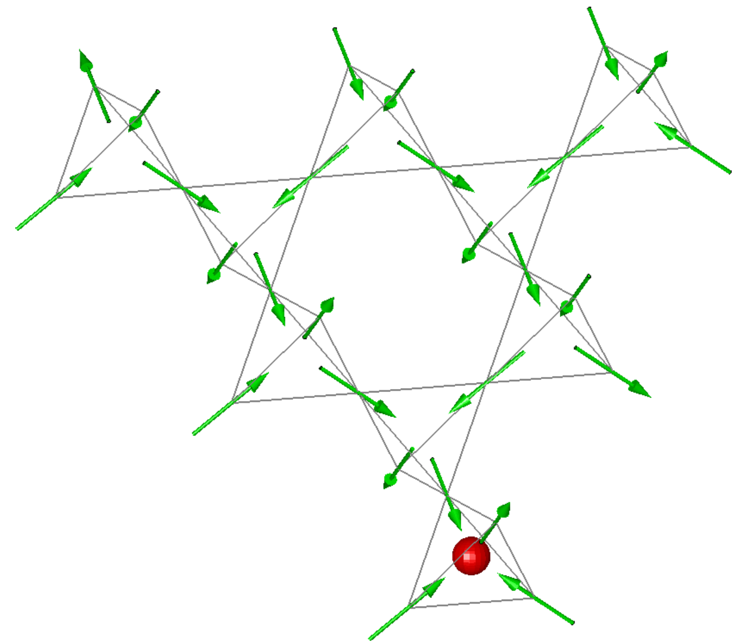
$$Q_m = -\int \mathbf{M} \cdot d\mathbf{S} = \int \mathbf{H} \cdot d\mathbf{S}.$$

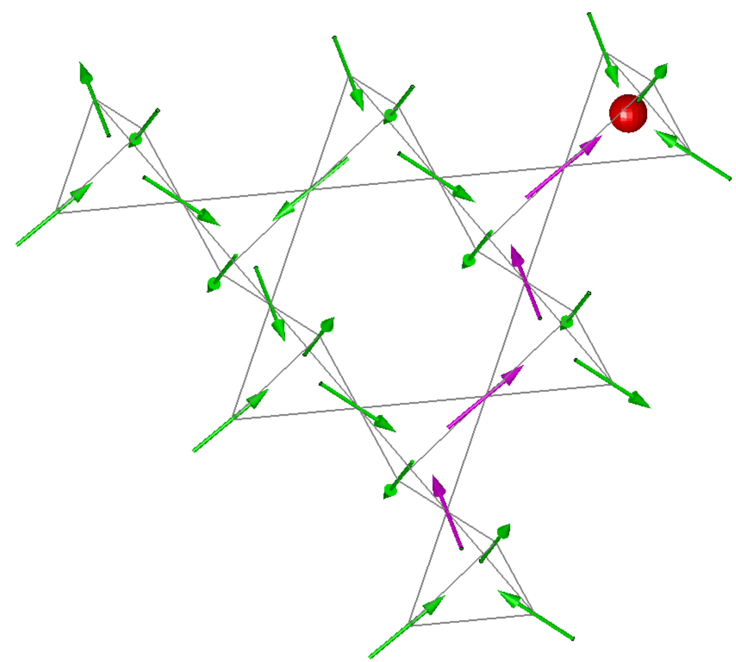


$Q_m = 0.$

$Q_m = -1;$
$\Delta E = J/2.$

$Q_m = 1;$
$\Delta E = J/2.$





Ivan Ryzhkin, JETP 2005.
C. Castelnovo, R. Moessner, & S. Sondhi, Nature 2008.

Quantum spin ice

- Substantial quantum fluctuations.
- Quantum tunneling of magnetic charges.
- Classical magnetic charges ➔ Spinons
- Spinons are responsible for many physical properties of QSI. But how does the spinon move?

QSI Survey: M. Gingras and P. McClarty, Rep. Prog. Phys. 2014.

nature physics

ARTICLES

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A measure of monopole inertia in the quantum spin ice $Yb_2Ti_2O_7$

LiDong Pan[1], N. J. Laurita[1], Kate A. Ross[1,2], Bruce D. Gaulin[3,4,5] and N. P. Armitage[1*]

ARTICLE

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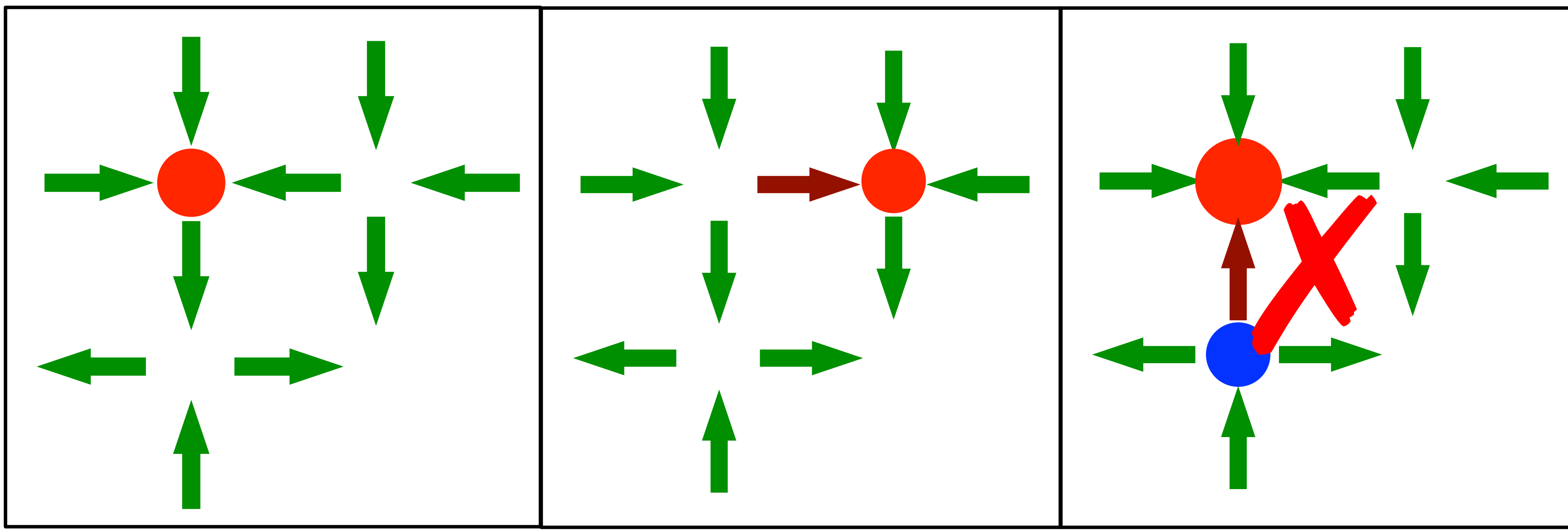
Possible observation of highly itinerant quantum magnetic monopoles in the frustrated pyrochlore $Yb_2Ti_2O_7$

Y. Tokiwa[1,2,*], T. Yamashita[1,*], M. Udagawa[3], S. Kittaka[4], T. Sakakibara[4], D. Terazawa[1], Y. Shimoyama[1], T. Terashima[2], Y. Yasui[5], T. Shibauchi[6] & Y. Matsuda[1]

Single-spinon dynamics

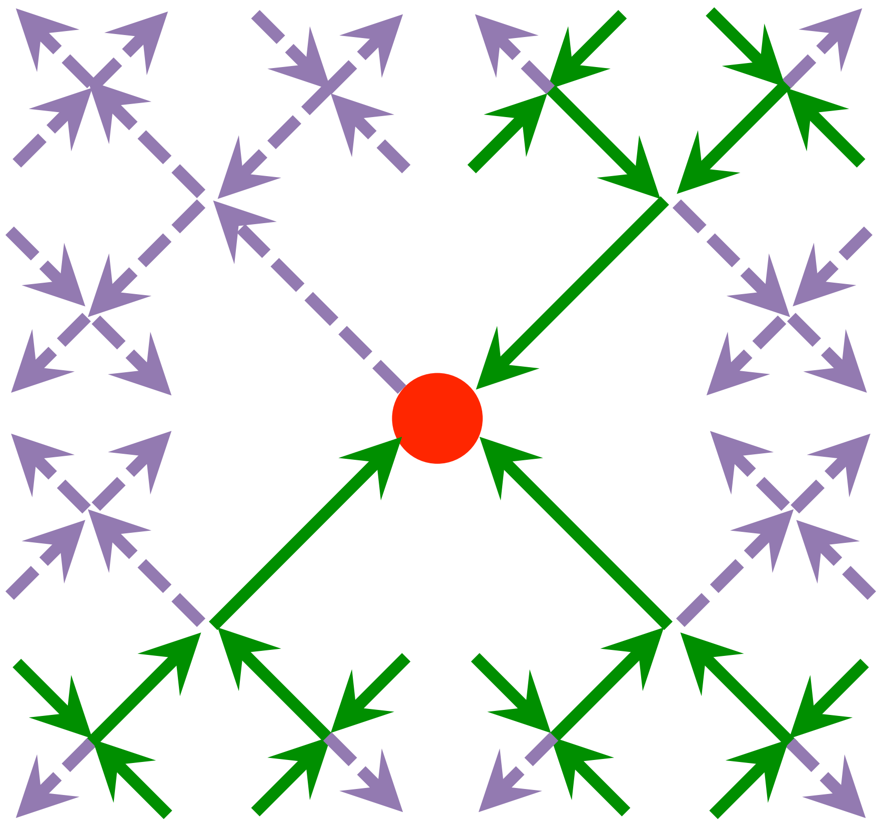
Hardcore Bosons

$$H = -\sum_{\langle ij\rangle}\left(|\underset{i}{\oplus} \leftarrow \underset{j}{\cdot}\rangle\langle \underset{i}{\cdot} \rightarrow \underset{j}{\oplus}| + |\underset{i}{\ominus} \rightarrow \underset{j}{\cdot}\rangle\langle \underset{i}{\cdot} \leftarrow \underset{j}{\ominus}|\right)$$



M. Chen, L. Onsager, J. Bonner, and J. Nagle, JCP, 1974.
O. Petrova, R. Moessner, S. Sondhi, PRB, 2015.

Spinon walks on a tree



M. Chen, L. Onsager, J. Bonner, and J. Nagle, JCP, 1974.
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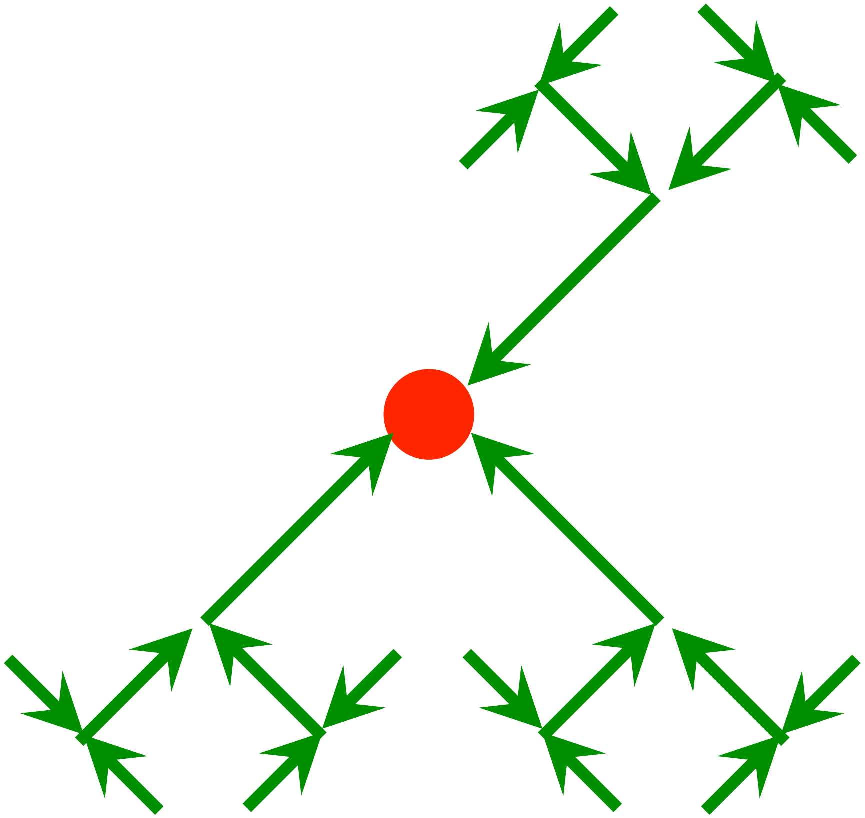
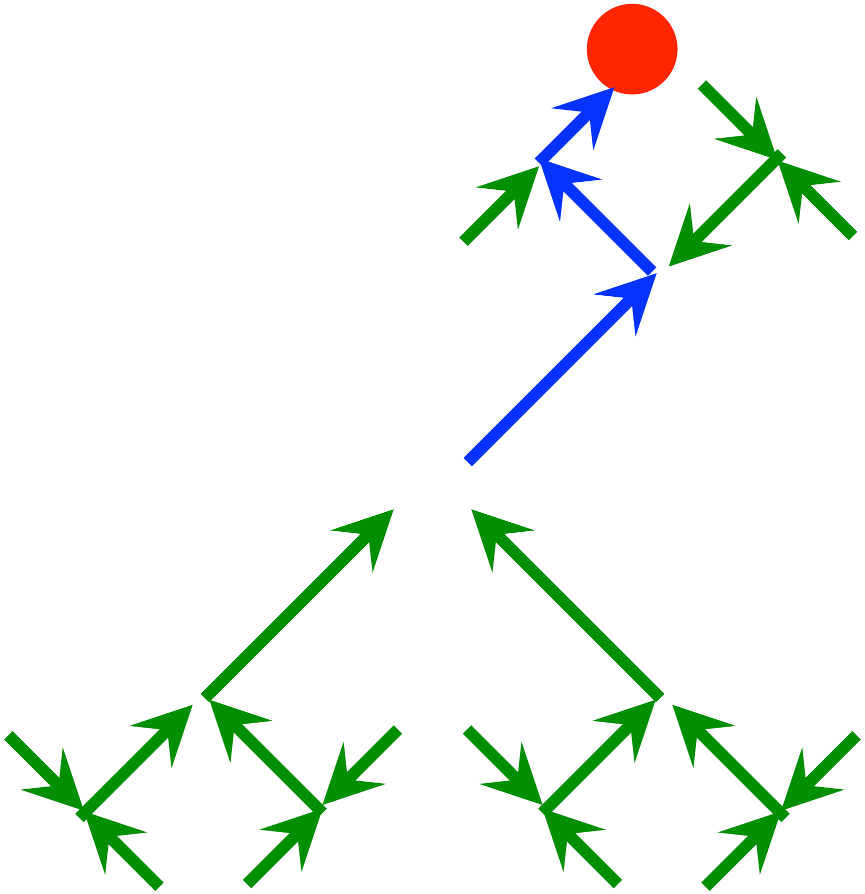
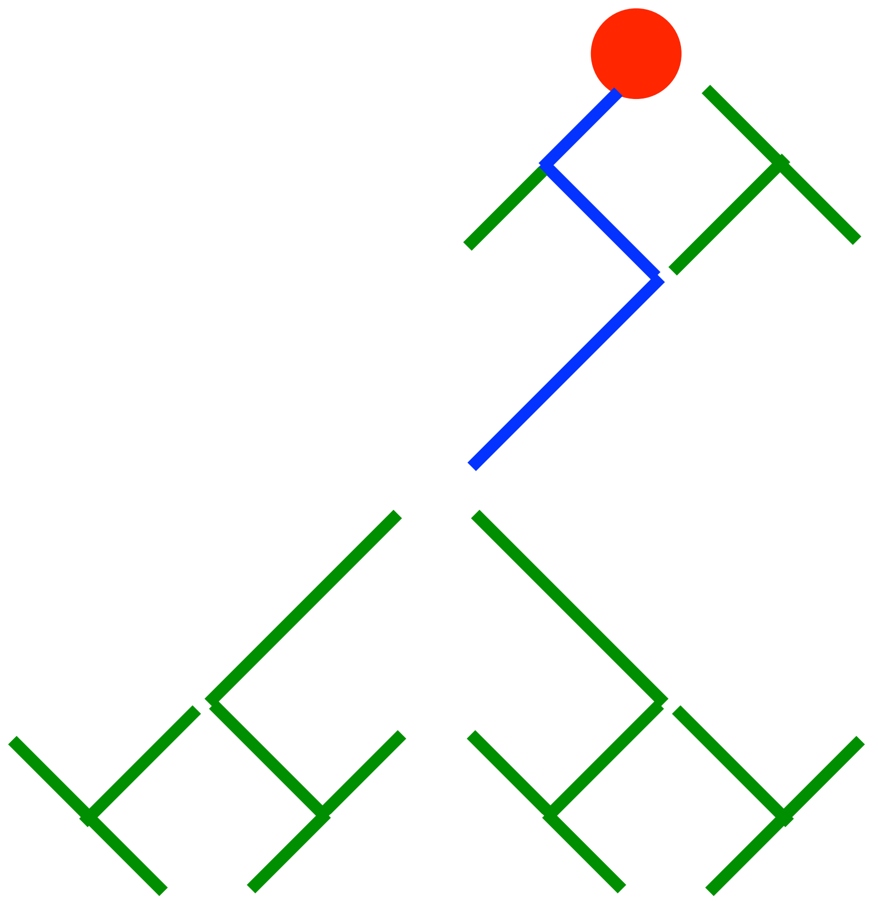
Spinon walks in a tree



Spinon walks in a tree

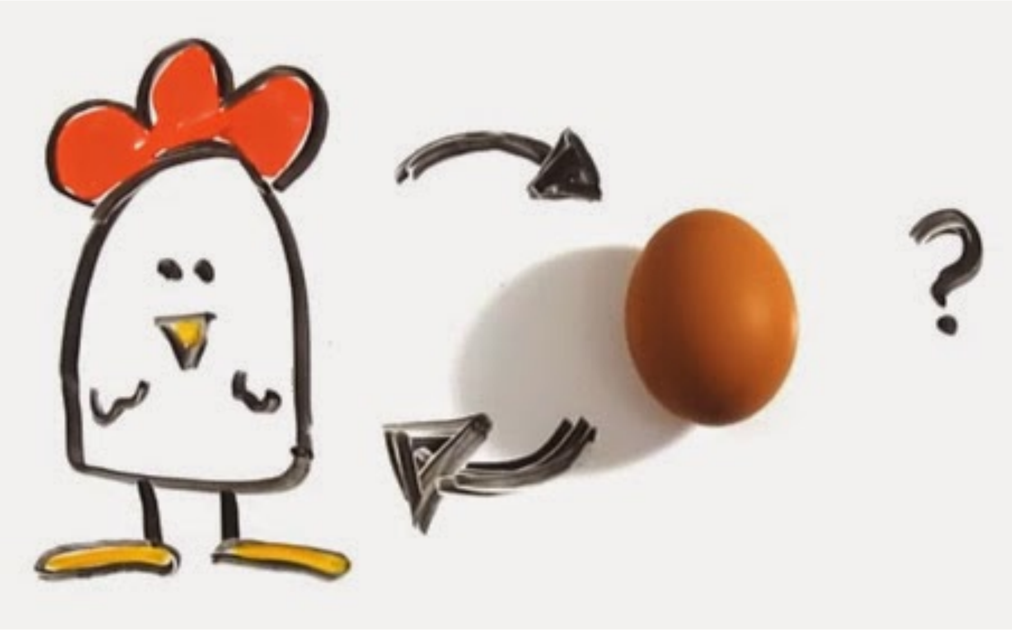


Spinon walks in a tree



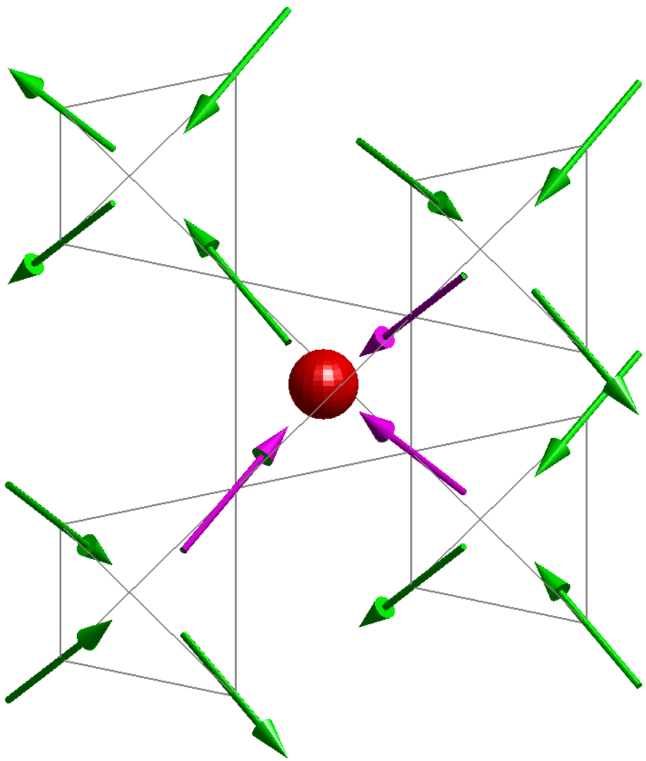
A chicken and egg problem

- Background spins guide spinon motion.
- Lattice contains loops.
- New spin background each time spinon revisits a site.



Ground State

$$\sum_{\alpha} \langle \alpha | H | \beta \rangle = -3. \quad \Rightarrow \quad |\text{G.S.}\rangle = \sum_{\alpha} |\alpha\rangle$$



$$\begin{aligned} H|\text{G.S.}\rangle &= \sum_{\alpha} H|\alpha\rangle \\ &= \sum_{\alpha,\beta} |\beta\rangle\langle\beta|H|\alpha\rangle \\ &= -3\sum_{\beta}|\beta\rangle \\ &= -3|\text{G.S.}\rangle. \end{aligned}$$

Characterizing single-spinon dynamics

$$C_{ij}(t) = \frac{\langle \mathrm{G.S.}| e^{i\hat{H}t} \hat{n}_j e^{-i\hat{H}t} \hat{n}_i |\mathrm{G.S.}\rangle}{\langle \mathrm{G.S.}|\hat{n}_i|\mathrm{G.S.}\rangle} \overset{it\to\tau}{\Rightarrow} C_{ij}(\tau) = \frac{\langle \mathrm{G.S.}| e^{\hat{H}\tau} \hat{n}_j e^{-\hat{H}\tau} \hat{n}_i |\mathrm{G.S.}\rangle}{\langle \mathrm{G.S.}|\hat{n}_i|\mathrm{G.S.}\rangle}$$

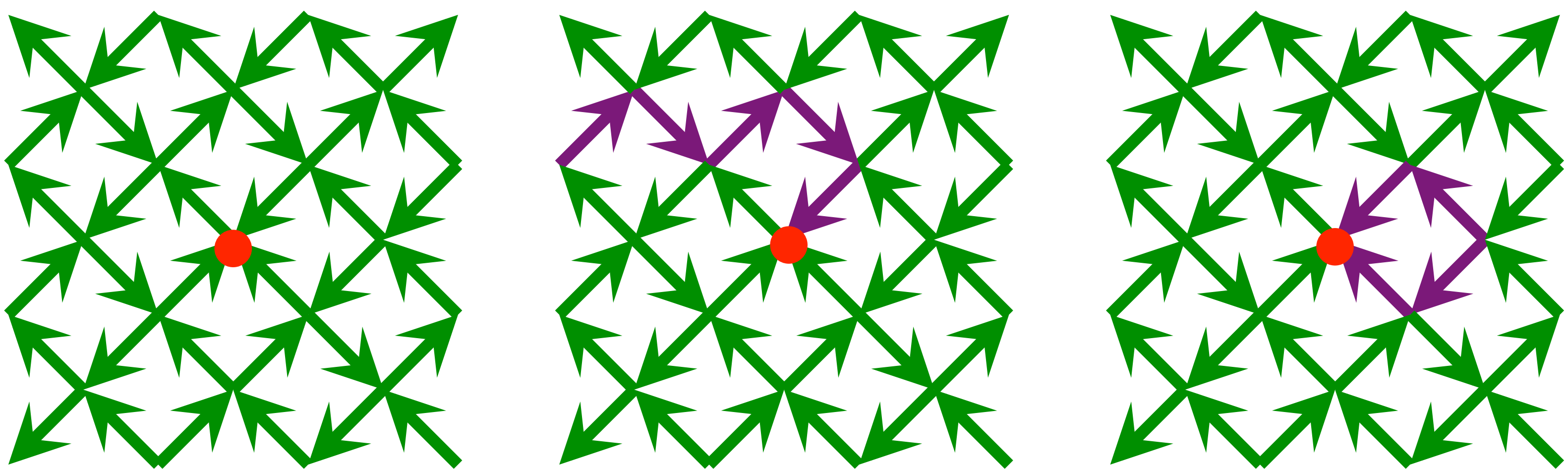
$\boxed{C_{ij}(t)}$ Probability of observing the spinon on site j at time t provided it was observed on i at time 0.

Spinon path integral

$$C_{ij}(\tau) \propto \frac{1}{\sum_{\alpha} 1} \sum_{\alpha} \sideset{}{'}\sum_{\gamma:(i,0)\to(j,\tau)} (\delta\tau)^{L_{\gamma}}$$

γ: All paths that are allowed by initial spin config. α.

α. γ. ✔ γ. ✗

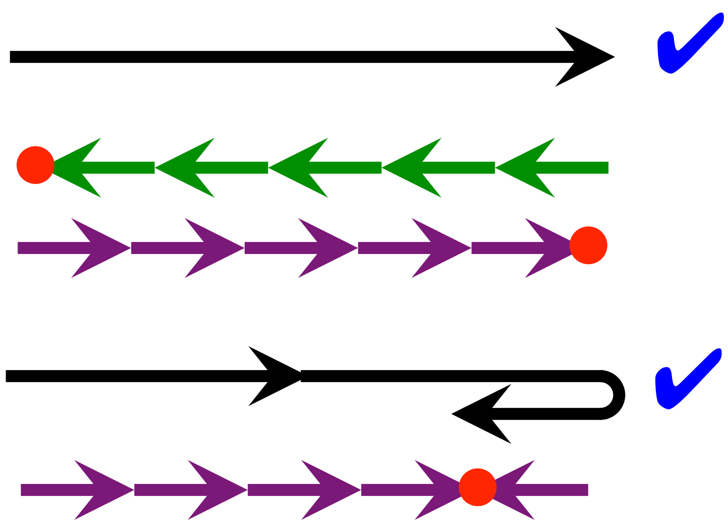


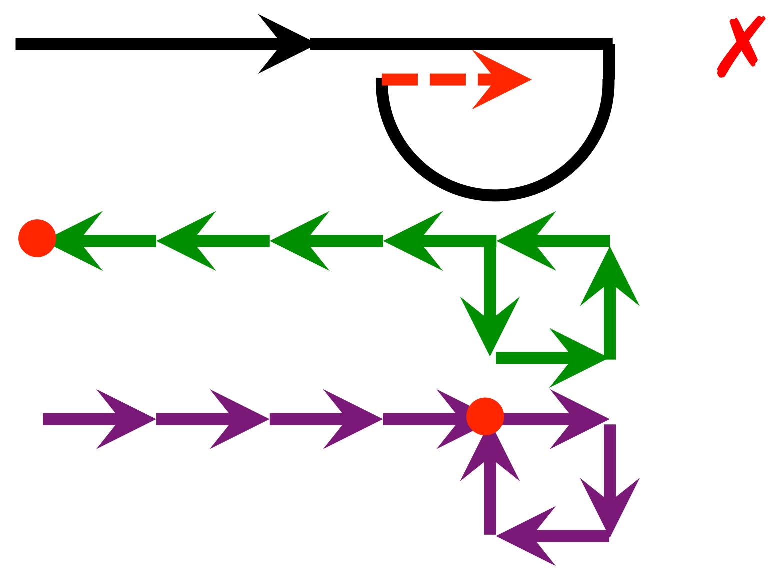
Trace out spins, acquire memory

$$C_{ij}(\tau) \propto \frac{1}{\sum_\alpha 1} \sum_\alpha \sideset{}{'}\sum_{\gamma:(i,0)\to(j,\tau)} (\delta\tau)^{L_\gamma} = \sum_{\gamma:(i,0)\to(j,\tau)} W_\gamma (\delta\tau)^{L_\gamma}$$

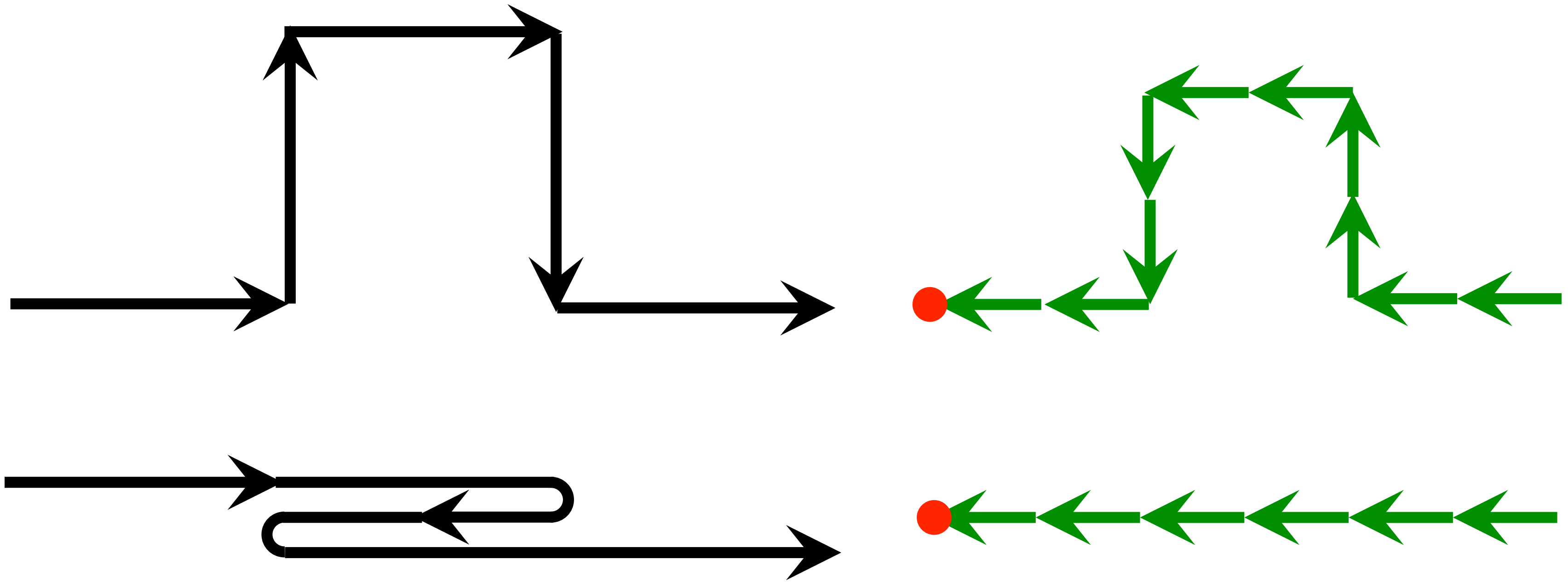
$$W(\gamma) = \frac{\text{Number of initial spin states for which } \gamma \text{ is feasible}}{\text{Number of all initial spin states}}$$

$$= e^{\text{Entropy cost of } \gamma}.$$



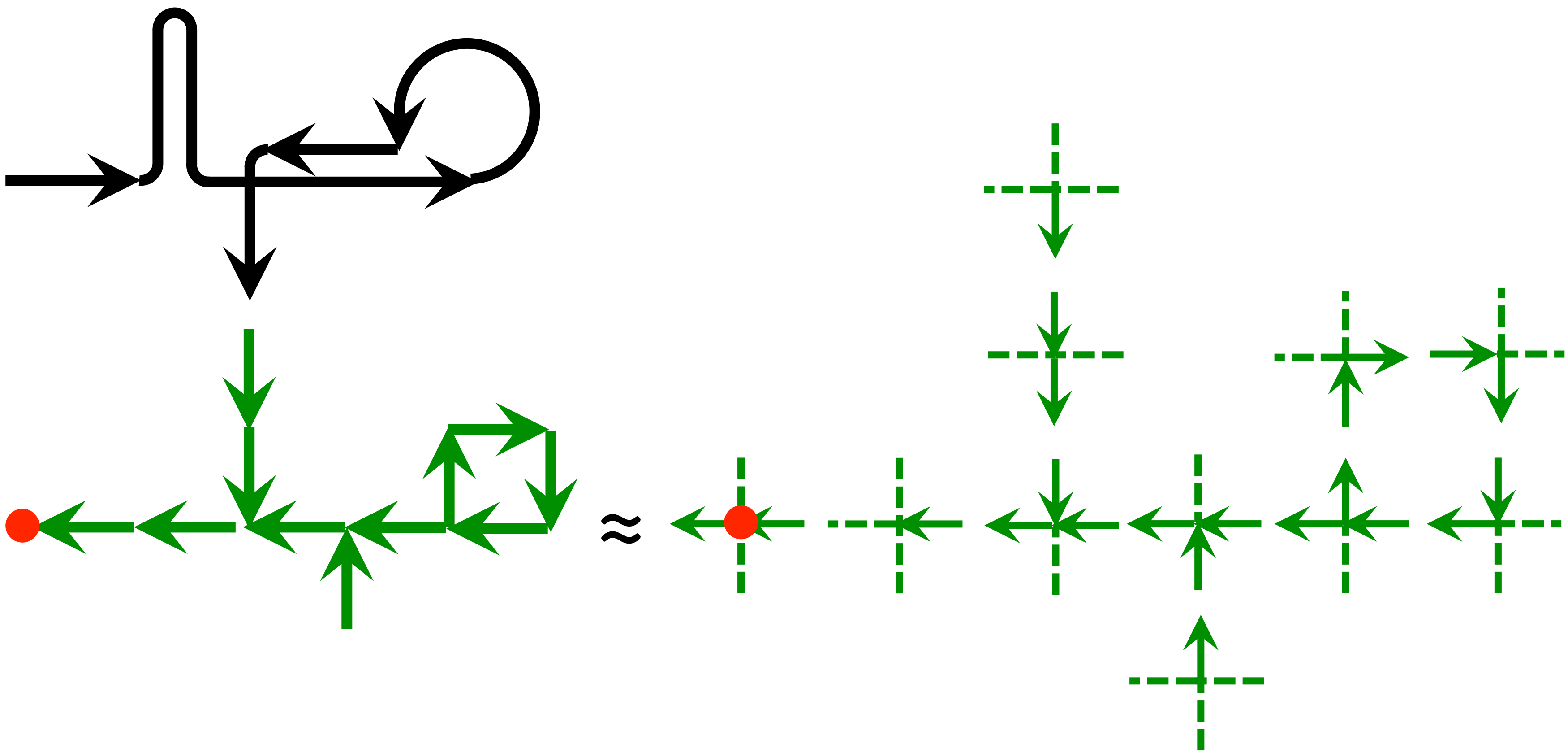


Spinon likes retracing its steps.



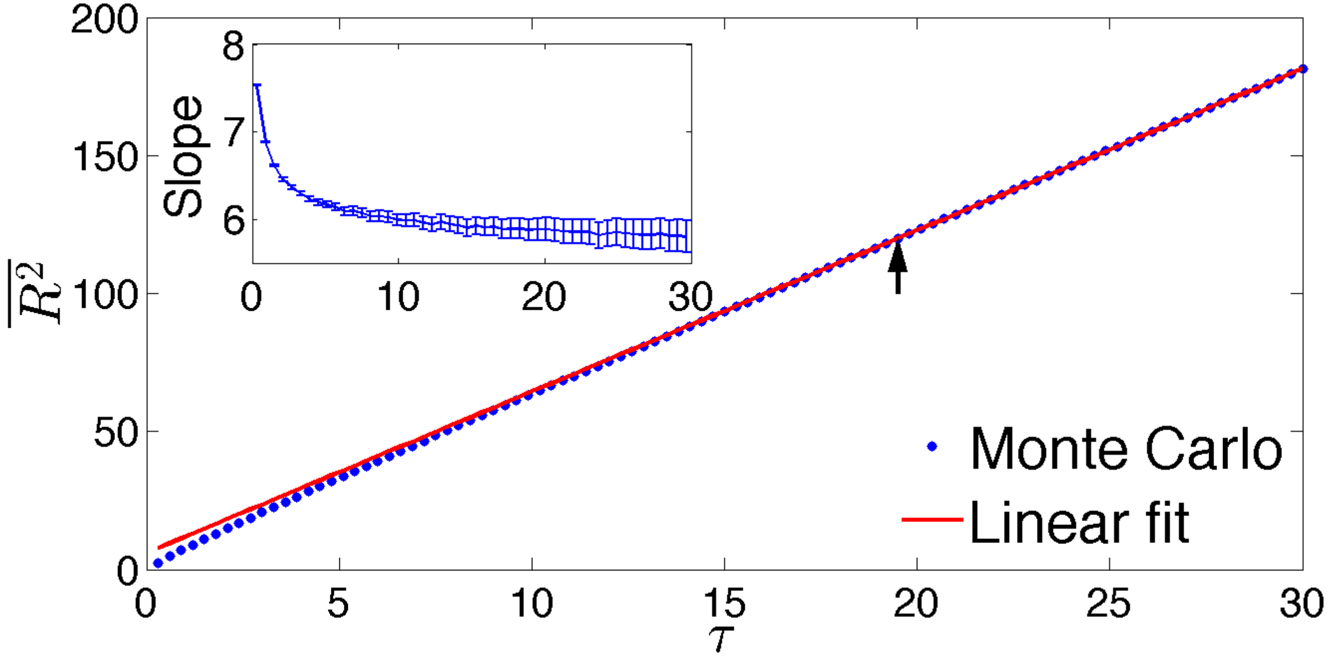
Estimating $W(\gamma)$

$$C_{ij}(\tau) = \sum_{\gamma:(i,0)\to(j,\tau)} W_\gamma (\delta\tau)^{L_\gamma} / (\sum_{\gamma:(i,0)} W_\gamma (\delta\tau)^{L_\gamma})$$



Mean Displacement Squared

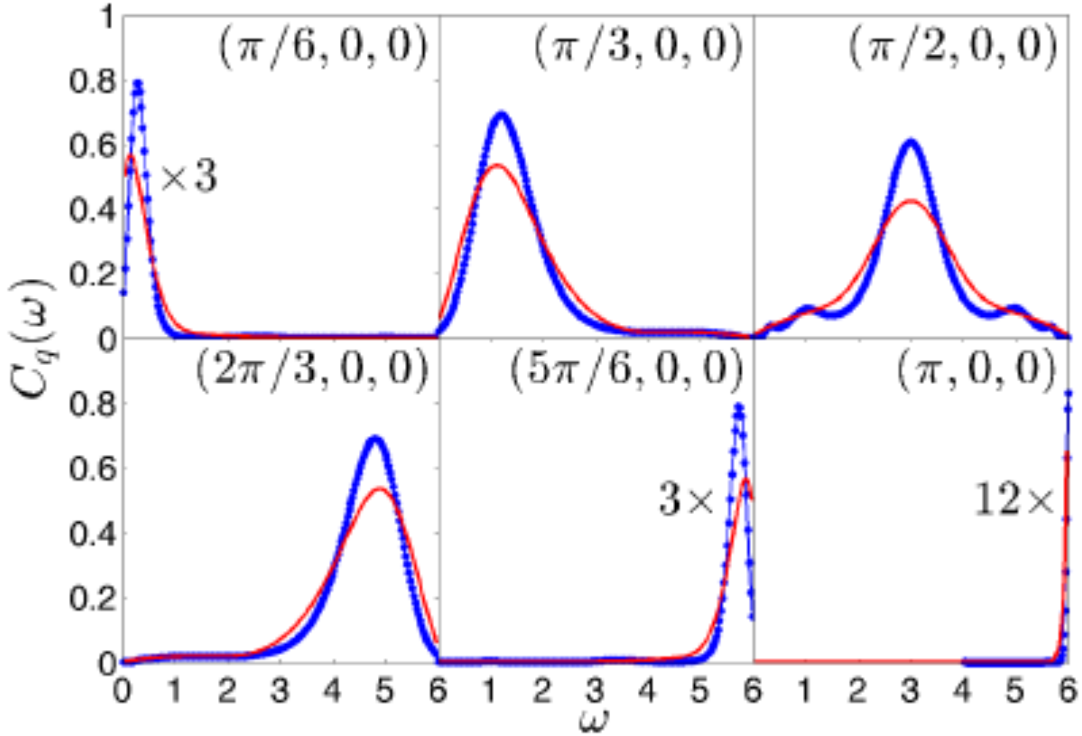
$$m_{\text{eff}} = 0.51 > 2m_{\text{tight binding}}$$



$$i\frac{\partial \psi}{\partial t} = -\frac{1}{2m}\nabla^2\psi \Rightarrow \frac{\partial \psi}{\partial \tau} = \frac{1}{2m}\nabla^2\psi$$

"Diffusion Constant": $D = \dfrac{1}{2m}.$

$$C_{\mathbf{q}}(\omega) = \int dt \sum_j C_{0j}(t) e^{-i(\mathbf{q}\cdot\mathbf{R}_{0j} - \omega t)}$$



$$\text{Compare} : C_{\mathbf{q}}(\omega) \propto \delta(\omega - E_{\mathbf{q}} + E_{\mathbf{q}=0})$$

Outlook

- Spinon dynamics is (infinitely) strong-coupled at lattice scale.
- Spinon behaves as a nearly-free, massive particle at low energy.
- Incoherent spin background?